A Model of State Aggregation*

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Optimizing utility in all states of the world together might be difficult even for a machine. This paper adds to the behavioral literature by introducing a model in which the agent aggregates the states together, even though she is aware of the entire state space. As a result of the state aggregation, the person solves several problems with fewer variables instead of the initial problem with the entire state space. When the person is an SEU-maximizer, the decisions are not affected by the way the states get aggregated. In our model, people still have subjective priors over states and events, however, they lump some states together in a non-linear way, which leads to different choices. The paper provides axioms for a state aggregation model, discusses identification of the state aggregation from choices in a complete market setting, offers comparative statics due to changes in the state aggregation and event aversion, and demonstrates how the model explains a number of ambiguity paradoxes.

Keywords: state aggregation, decision under risk, identification, SEU, ambiguity paradoxes, axioms

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1 Introduction

Economists usually assume that people have perfect computational abilities and a good understanding of how the world works. However, in reality, optimizing utility in all states of the world at the same time might be difficult even for a machine. The economics literature (e.g., Simon (1955), Akerlof and Yellen (1985), Houser, Keane, and McCabe (2004), Carvalho and Silverman (2017)) provides helpful insights into different ways that people might simplify decision-making. This paper adds to the discussion by introducing a model in which the agent aggregates the states together, even though she is aware of the entire state space. There are several possible reasons for doing this, including settings with a large number of states or states that are similar to one another. As a result of state aggregation, a person solves several problems with fewer variables instead of the initial problem with the entire state space. When the person is a Subjective Expected Utility (SEU)-maximizer, her decisions are not affected by the way the states get aggregated. However, a vast literature (Ellsberg (1961), Kahneman and Tversky (1979), Halevy (2007), etc.) demonstrates that SEU does not describe peoples' choices in practice. We offer a model that is a slight deviation from SEU: people still have subjective priors over states and events, however, they lump some states together in a non-linear manner. We obtain that various methods of state aggregation lead to different choices. For this reason, as this paper demonstrates, the state aggregation of each individual might be identified from observed choices.

To illustrate the idea of state aggregation, consider the following example: Imagine a person who has just bought a car and needs to choose an insurance plan. There are three states of the world – an accident with the agent at fault, an accident with someone else at fault, and a natural disaster. An insurance policy consists of three corresponding deductibles. First, suppose that the agent is a person who does not aggregate states. Generally, her choice is a bundle of deductibles such that her consumption is smoothed between all three states. Next, imagine a person sitting in front of an insurance agent and reading through the book with descriptions of insurance plans and conditions. The book is difficult to read: it is written in a small font and includes too many conditions. Making a thorough choice is too complicated. The person clearly understands the state "accident with the agent at fault," however, "accident with the other party at fault" and "natural disaster" are not that different from the agent's perspective – in both of these cases it is not the agent's fault. Thus, in order to simplify her decision-making, the person combines collision and natural disaster into event "not my fault." The presence of the state aggregation changes the way the agent thinks about the world: instead of all three states, the state space is partitioned into aggregated events "my fault" and "not my fault." Moreover, the individual generally wants to smooth her consumption in two ways: (1) between her aggregated events, and (2) between the states inside the events. We show that a risk- and event-averse person makes riskier choices under more state aggregation due to more attention being paid to values at the aggregated events level rather than to the states at those events. As a result, the choice of deductibles will differ under various state aggregations, implying that state aggregation might be identified from choices.

In most economic situations, researchers treat the decision-making process like a black box, and all differences in behavior are usually explained by differences in preferences or information. This paper allows for situations in which agents with the same information and preferences might make different choices due to heterogeneity in state aggregation. Moreover, such behavior cannot be obtained by providing the agent with incomplete information, which is present in the model in the form of subjective probabilities of the states. Incomplete information under no state aggregation would imply smoothing consumption between all states given some priors. Choices in our model under non-trivial state aggregation do not satisfy consumption-smoothing across all states with any subjective probabilities.

Note that state aggregation can be interpreted as a frame in Salant and Rubinstein (2008). The authors point out that "real-life behavior often depends on observable information, rather than the set of feasible alternatives, which is irrelevant in the rational assessment of the alternatives but nonetheless affects behavior." One of the examples they use is a voter that may be influenced by the order of candidates on a ballot. They call such additional information a frame. In this paper, an insurance company decides how to formulate terms and conditions based on which states should be grouped in which section, the order of the sections, what information should be in the footnote, etc. This kind of information (frame) should be irrelevant to the rational agent. However, in our model, it defines state aggregation.

The notion of state aggregation is similar to intermediate information. Li (2011) introduced the concept of intermediate information: information that arrives after a choice has been made and before an outcome is realized. The author axiomatizes preference relations over pairs of acts and intermediate information together. In order to obtain such preference relations over pairs, Li derives preference relations between acts under some specific information by using conditional preferences and basic "no information" preferences as primitives. State aggregation can be interpreted as fixed intermediate information. However, it requires different axiomatization because the observable primitives are different.

The model's representation is also ideologically close to the smooth ambiguity of Klibanoff, Marinacci and Mukerji (2005) as well as SPS-EU of Ergin and Gul (2009) due to non-linear two-stage evaluation of the value functional. Smooth ambiguity and SPS-EU postulate that agents have second-order beliefs about the "right" distribution. Hence, the individual evaluates the expected utility under each possible distribution, and then applies the second-order beliefs with some curvature to obtain the final value. In our model, the consumer also evaluates expected utility, however, on the events with singleton priors. Later on she also applies some curvature, but while using probabilities of the events. However, note that smooth ambiguity and SPS-EU describe behavior under ambiguity. In contrast, our model talks about decision under risk.

Additionally, the idea of state aggregation is similar in spirit to Gul, Natenzon and Pesendorfer (2014). The authors consider an agent who combines similar objects together then chooses a group, and only then does she select an object from that group. In contrast, our agent combines states instead of objects.

Undoubtedly, understanding state aggregation is important for researchers and policy-makers. It is easy to imagine a situation in which an insurance company might be interested in complicating terms and conditions to nudge agents to aggregate states in a desired way. A social planner might want to prevent the insurance company from doing so by restricting a set of insurance plans.

In this paper, we (1) offer a model of state aggregation and axioms that define the representation of preferences; (2) identify state aggregation from observable choices; (3) provide comparative statics for agents with different event-aversion and for the case when two events get aggregated; and, (4) demonstrate how the model explains a number of ambiguity paradoxes.

First, we provide axioms and the representation of preferences with state aggregation. We assume that only preferences over acts are observed. Thus, our primitive is preferences over acts under some state aggregation from which we derive conditional preferences. We define the state aggregated restriction of Savage's Sure Thing Principle, which holds only on aggregated events instead of all states. The last restriction guarantees that the conditional preferences on the events in the state aggregation are well-defined. After that, we provide a general representation of preferences. Note that the representation is recursive in its nature: the value on events is defined in the first (conditional) stage, and later the value of the act (ex-ante stage) is a function of the values of the events. Then we add the corresponding versions of the independence axiom for ex-ante and conditional stages in order to describe the simplest non-SEU model of the state aggregation. We assume that the conditional functional that represents the value at each event is a positive non-affine transformation of some SEU over the states in each event. Furthermore, the ex-ante functional is just an expected value of the events. We call this model State Aggregation SEU (SASEU).

Second, we provide comparative statics in two cases. We start by investigating how the concavity of the curvature function that is associated with event aversion affects choices. We find that a more event-averse consumer smooths consumption across events more, and, as a result, smooths consumption across states more. Hence, the behavior of a more event-averse consumer looks less risky, which is a predictable result. Likewise, we study the effect of changes in state aggregation – in particular, how combining two events affects consumption choices. An interesting property holds: whenever something changes in state aggregation, consumption in all unaffected events and states moves in the same direction. When two events get aggregated, the absence of event aversion between these events allows the event values to get away from each other. This implies that the consumption at the states of the event with the greater value will increase even more, while consumption at the states of the event with the lower value will decrease. The opposite effects will also be present when an event gets split in two. On one hand, these implications could be used by an insurance company or portfolio manager to increase their profit. On the other, the social planner could use the results to prevent undesirable behaviors or nudge desirable ones.

Third, we show identification of the unobservable state aggregation, priors, and utility from choices and prices in a complete market under SASEU. We start by using first-order conditions to determine state aggregation. Note that SASEU implies SEU inside each event, and, hence, produces the same MRS when obtained for the states inside the same event. However, when MRS is calculated for two states with different events, it differs from SEU by including additional variables. This leads to identification of each event, and, thus, the state aggregation. The rest is standard: (1) SEU inside each event (conditional stage) provides with conditional probabilities and utility up to affine transformation; and, (2) SEU over events (ex-ante stage) delivers probabilities of the events together with the transformation applied to the conditional value.

Finally, SASEU offers an alternative explanation of ambiguity paradoxes suggested in Ellsberg (1961), Machina (2009) and Machina (2014). Halevy (2007) demonstrates empirical evidence for the relationship between the non-reduction of compound lotteries and ambiguity aversion. In our model, there is no need for ambiguity or multiple priors to produce the phenomena. The agent has reasonable subjective probabilities of the states. Nevertheless, aggregation of ambiguity states together and the curvature in the conditional functional contradicts the reduction of compound lotteries. That is why, similar to Dillenberger and Segal (2015), our model is able to explain different ambiguity paradoxes.

Section 2 introduces the model and demonstrates how it explains ambiguity paradoxes while also providing comparative statics results coupled with some examples. Section 3 discusses the identification of state aggregation from choices and prices in the complete market setting. Axioms and the representation results are in Section 4. All proofs are provided in the Appendix.

2 State Aggregation SEU

2.1 The Model

Suppose that X is a compact convex subset of consequences in $\mathbb{R}$, and Ω is a finite set of states of the world with an algebra Σ of subsets of Ω. We denote $\mathcal{F}$ as a set of all acts, Σ-measurable finite step functions: $\Omega \to \Delta X$. Let $\mathcal{M}$ be a set of elements π, such that $\pi \subset \Sigma$ is a partition of Ω. Partitions of the state space represent different ways of state aggregation.

We denote for all $x, y \in \mathcal{F}, A \in \Sigma$, xAy an act: $xAy(s) = x(s)$ if $s \in A$, and $xAy(s) = y(s)$ if $s \notin A$. The mixture of acts is defined statewise. We will abuse notation and define X as a set of constant acts in what follows below.

Definition 1. *$\succeq$ is said to exhibit SEU representation if and only if there exist probabilities $P(s)$ for any state $s \in \Omega$, and a continuous monotone function $u : X \to \mathbb{R}$*

such that

$$x \succeq y \Leftrightarrow \sum_{s \in \Omega} P(s) u(x(s)) \geq \sum_{s \in \Omega} P(s) u(y(s)).$$

The above definition is the standard SEU model. Our innovation is the State Aggregation Subjective Expected Utility (SASEU) model. We postulate that the decision-maker aggregates states into a partition π of Ω. Our ultimate goal is to provide an identification procedure for state aggregation π. The introduction of state aggregation leads to the following extension of SEU.

Definition 2. $\succeq$ *is said to exhibit SASEU representation if there exist a partition of the state space* π*, probabilities* $P(s|A)$ *and* $P(A|\pi)$ *for any state* $s \in A \in \pi$*, a continuous monotone function* $u : X \to \mathbb{R}$*, and an increasing function* $\phi : \mathbb{R} \to \mathbb{R}$ *such that*

$$x \succeq y \Leftrightarrow \sum_{A \in \pi} P(A) \phi \left(\sum_{s \in A} P(s|A) u(x(s)) \right) \geq \sum_{A \in \pi} P(A) \phi \left(\sum_{s \in A} P(s|A) u(y(s)) \right).$$

A set of axioms for the above representation is provided in Section 4.

Definition 2 implies that the value functional of the consumer under SASEU is defined in a recursive manner and consists of two stages. First, the conditional stage is when the agent aggregates states into event A and evaluates the act x given every such event A in the partition π:

$$V_A(x) = \phi \left(\sum_{s \in A} u(x(s)) P(s|A) \right),$$

where $P(s|A)$ is the conditional probability of state s given event A. Second, the ex-ante stage is when the agent evaluates the act across events that form her state aggregation.

$$V_\pi(x) = \sum_{A \in \pi} V_A(x) P(A),$$

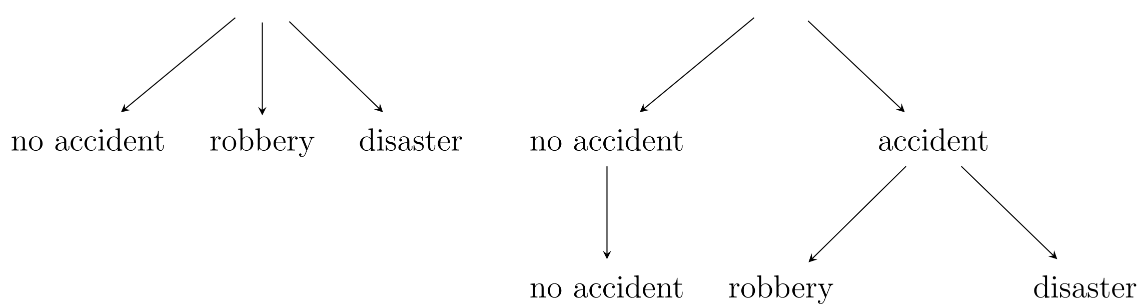
where $P(A)$ is the probability of event A.

2.1.1 SASEU vs. SEU

Notice that we assume that $V_\pi(x) = \sum_{A \in \pi} P(A) V_A(x)$ is a regular SEU functional, while $V_A(x) = \phi\left(\sum_{s \in A} P(s|A) u(x(s))\right)$ is a positive transformation of a conditional stage SEU functional. If either $\phi(\cdot)$ is linear or $\pi = \Omega$, then the model simplifies to the regular SEU in Definition 1. To understand the difference between two models, consider the following example.

The agent chooses a home insurance policy under three states of the world: natural disaster, robbery, and no accident.

Suppose that the probability of no accident (s_1) is 90%, the probability of robbery (s_2) is 7%, and the probability of natural disaster (s_3) is 3%. Imagine two different situations: (1) the agent does not aggregate states, and her state space is $\pi_0 = \Omega = \{s_1, s_2, s_3\}$; and (2) the agent has difficulty optimizing over three states, and she splits the whole outcome space into events "no accident" $A_1 = \{s_1\}$ and "accident" $A_2 = \{s_2, s_3\}$. We denote the state aggregation in this case as $\pi = \{A_1, A_2\}$.



In situation (1), the agent is a regular SEU-maximizer and the ex-ante stage implies regular evaluation over the whole state space. Thus, the agent's functional is

$$V_{\pi_0}(x) = 0.9\phi(u(x_1)) + 0.07\phi(u(x_2)) + 0.03\phi(u(x_3)),$$

where x_i denotes consumption in state s_i.

In situation (2), the agent's state aggregation π consists of two events, A_1 and A_2. In addition, consider that the agent updates priors using Bayes' rule. Thus, the set of ex-ante priors is $P(A_1) = 0.9$ and $P(A_2) = 0.1$. In the case of no accident, the conditional on event A_1 probability is degenerate: $P(s_1|A_1) = 1$. If the event is "accident," then the conditional on event A_2 probabilities are $P(s_2|A_2) = 0.7$ and $P(s_3|A_2) = 0.3$. Then the value functional is

$$V_\pi(x) = 0.9\phi(u(x_1)) + 0.1\phi\,(0.7u(x_2) + 0.3u(x_3)\,).$$

As long as $\phi(\cdot)$ is not linear, the maximization of $V_{\pi_0}(x)$ and $V_\pi(x)$ will result in different solutions.

2.1.2 SASEU vs. Two-Stage Lotteries

Recursive evaluation in SASEU makes it closely related to two-stage lotteries when the reduction axiom does not hold. Indeed, if the space of two-stage lotteries is restricted to π-measurable lotteries, then SASEU will be equivalent to Expected Utility without Time Neutrality in Segal (1990)[1].

Consider again $\pi = \{A_1, A_2\}$, where $A_1 = \{s_1\}$ and $A_2 = \{s_2, s_3\}$. Suppose the agent is offered a two-stage lottery $X = (X_1, 0.9; X_2, 0.1)$, where X_i is a lottery on A_i, i.e., $X_1 = (x_1, 1)$ and $X_2 = (x_2, 0.7; x_3, 0.3)$. In this case, the representation in the case of SASEU and Segal's example of EU3 is identical:

$$V_\pi(X) = 0.9\phi(u(x_1)) + 0.1\phi\,(0.7u(x_2) + 0.3u(x_3)\,).$$

However, suppose the agent still aggregates states into π but she is offered a two-stage lottery that is not π-measurable. Consider a lottery Y that pays a lottery Y_1 if s_1 or s_2 happens (though we do not know which) and a payoff y_3 if s_3 realizes, where Y_1 is a lottery that pays y_1 if s_1 occurs, and y_2 in the case of s_2. Hence, Segal's EU3

[1]See Example 2 in Segal (1990).

would imply the following representation

$$V_{EU3}(Y) = 0.97\phi\left(\frac{90}{97}u(y_1) + \frac{7}{97}u(y_2)\right) + 0.03\phi\left(u(y_3)\right),$$

while SASEU would still require aggregation into π

$$V_\pi(X) = 0.9\phi(u(y_1)) + 0.1\phi\left(0.7u(y_2) + 0.3u(y_3)\right).$$

2.1.3 SASEU vs. Smooth Ambiguity

Ambiguity might provide one of the most natural ways to make a consumer to aggregate states, while still allowing for a subjective singleton prior about probabilities. Consider the following example. The state space $\Omega = \{s_1, s_2, s_3\}$ and it is known that the probability of s_1 is $\frac{1}{3}$. Nothing is known about the probabilities of other states. Uncertainty about the probabilities of s_2 and s_3, and the fact that they sum to $\frac{2}{3}$ might nudge the agent to aggregate them together. We denote event $A = \{s_2, s_3\}$. Given the symmetry of the situation, there is no reason for the consumer to believe that the probability of s_2 is greater or smaller than s_3. Hence, $P(s_2|A) = P(s_3|A) = 0.5$. In this case, the agent evaluates an asset x that delivers x_i in state s_i in the following way:

$$V_{SASEU}(x) = \frac{1}{3}\phi(u(x_1)) + \frac{2}{3}\phi(0.5u(x_2) + 0.5u(x_3)).$$

Two-stage SEU evaluation makes SASEU ideologically similar to the smooth ambiguity of Klibanoff, Marinacci and Mukerji (2005) as well as SPS-EU of Ergin and Gul (2009). However, the representation is substantially different. In these models, the agent has second-order beliefs about possible "right priors." For simplicity, assume that the agent believes that with equal probabilities the true probability of s_2

might be either $\frac{1}{6}$ or $\frac{1}{2}$. Then the consumer evaluates asset x in the following manner.

$$V_{SPS}(x) = \frac{1}{2}\phi\left(\frac{1}{3}u(x_1) + \frac{1}{6}u(x_2) + \frac{1}{2}u(x_3)\right) + \frac{1}{2}\phi\left(\frac{1}{3}u(x_1) + \frac{1}{2}u(x_2) + \frac{1}{6}u(x_3)\right).$$

Even if the agent has a singleton prior $P(s_2) = P(s_3) = \frac{1}{3}$ in the smooth ambiguity model, the representations do not get similar. It just simplifies to SEU in this case:

$$V_{SPS}(x) = \phi\left(\frac{1}{3}u(x_1) + \frac{1}{3}u(x_2) + \frac{1}{3}u(x_3)\right).$$

2.2 Comparative Statics

In this section, we discuss how changes in state aggregation might affect consumption.

Suppose there are S states of the world $\Omega = \{s_1, \ldots, s_S\}$. Consider an agent who chooses consumption $x = (x_1, \ldots, x_S)$ for every state of the world. Income is I, the price of 1 unit of consumption in state s is p_s, and the state probabilities are objective and denoted by $P(s)$.

Consider first an agent who does not aggregate states. Then the problem she solves is

$$\max_x \sum_{s\in\Omega} P(s)\phi(u(x_s))$$
$$\text{s.t.} \sum_{s\in\Omega} p_s x_s = I.$$

For any two states $s_1, s_2 \in \Omega$, the first-order condition is

$$\frac{p_{s_1}}{p_{s_2}} = \frac{P(s_1)}{P(s_2)} \frac{\phi'(u(x_{s_1}))}{\phi'(u(x_{s_2}))} \frac{u'(x_{s_1})}{u'(x_{s_2})}. \tag{1}$$

Now suppose that the agent aggregates states into π and updates probabilities according to the definition of conditional probability, – i.e., if $s \in A$ then $P(s) =$

$P(A)P(s|A)$. Hence, the agent's problem is

$$\max_x \sum_{A\in\pi} P(A)\phi\left(\sum_{s\in A} P(s|A)u(x_s)\right)$$
$$\text{s.t. } \sum_{s\in\Omega} p_s x_s = I.$$

If $s_1 \in A_1$ and $s_2 \in A_2$, then the first-order condition is

$$\frac{p_{s_1}}{p_{s_2}} = \frac{P(s_1)}{P(s_2)} \frac{\phi'\left(\sum_{s\in A_1} P(s|A_1)u(x_s)\right)}{\phi'\left(\sum_{s\in A_2} P(s|A_2)u(x_s)\right)} \frac{u'(x_{s_1})}{u'(x_{s_2})}. \tag{2}$$

Note that $\frac{u'(x_{s_1})}{u'(x_{s_2})}$ is present in both equations and is responsible for consumption smoothing due to risk aversion. However, $\frac{\phi'\left(\sum_{s\in A_1} P(s|A_1)u(x_s)\right)}{\phi'\left(\sum_{s\in A_2} P(s|A_2)u(x_s)\right)}$ is different. If $\phi(\cdot)$ is concave then the ratio produces consumption smoothing over events due to risk aversion over the events, and we call it event aversion. On the other hand, if $\phi(\cdot)$ is convex then the agent is risk-loving toward the events. However, we do not consider the convex case in this paper. Thus, in combination with consumption smoothing over states that comes from $u(\cdot)$, the state aggregation might have a different effect on choices made by the consumer.

To guarantee that the optimality conditions hold everywhere, we assume concavity:

Assumption 1. *$u(\cdot)$ and $\phi(\cdot)$ are strictly increasing twice differentiable concave functions.*

2.2.1 Event Aversion

First, we compare the choices of two individuals with different $\phi(\cdot)$. To do so, we use the following definitions.

Definition: *The function ϕ is* ***more concave*** *than $\tilde{\phi}$ if and only if there exists some increasing concave function g such that $\phi = g \circ \tilde{\phi}$.*

Definition: *For each income I, $u(\cdot)$, π, probabilities $P(s)$ and prices p_s $\forall s \in \Omega$, we define the* ***limit value*** $N(u, \pi, P, p, I)$ *such that for all $A \in \pi$ and all $s_i, s_j \in \Omega$*

$$N(u, \pi, P, p, I) = V_A(c) = \sum_{s \in A} P(s|A)u(x_s)$$
$$\sum_i p_i x_i = I$$
$$\frac{P(s_i)}{P(s_j)} \frac{u'(x_i)}{u'(x_j)} = \frac{p_i}{p_j}.$$

The limit value for each given problem represents the value a person with infinite event aversion achieves at each event. Such individual will always choose consumption such that the values of all events are equal to the limit value.

Theorem 1. *Suppose $\phi(\cdot)$ is more concave than $\tilde{\phi}(\cdot)$, while keeping income I, $u(\cdot)$, π, probabilities $P(s)$ and prices p_s $\forall s \in \Omega$ constant for both consumers. Denote by x and $\tilde{x}$ consumption bundles chosen by agents with ϕ and $\tilde{\phi}$ respectively. If Assumption 1 holds, then for any $A \in \pi$, one of the following is true*

1. $V_A(\tilde{x}) = \sum_{s \in A} P(s|A)u(\tilde{x}_s) < V_A(x) = \sum_{s \in A} P(s|A)u(x_s) < N(u, \pi, P, p, I) \Leftrightarrow \tilde{x}_s < x_s$ *for any $s \in A$;*

2. $V_A(\tilde{x}) = \sum_{s \in A} P(s|A)u(\tilde{x}_s) > V_A(x) = \sum_{s \in A} P(s|A)u(x_s) > N(u, \pi, P, p, I) \Leftrightarrow \tilde{x}_s > x_s$ *for any $s \in A$;*

3. $V_A(\tilde{x}) = \sum_{s \in A} P(s|A)u(\tilde{x}_s) = V_A(x) = \sum_{s \in A} P(s|A)u(x_s) = N(u, \pi, P, p, I) \Leftrightarrow \tilde{x}_s = x_s$ *for any $s \in A$.*

For each consumer the events can be split into two groups: those that have a value greater than the limit value and those with a lower value. The greater the event aversion (the more concave function ϕ is), the closer values will be to the limit value, implying that consumption in the events with lower values increases, while consumption in the events with higher values decreases. Note that it is possible to establish the opposite bound for consumption and event values – the opposite limit comes from solving the problem with a linear ϕ.

2.2.2 Aggregation and Disaggregation

The next theorem provides comparative statics for the situation in which one of the events is split into two in state aggregation. Note that combining two events would produce the opposite result.

Theorem 2. *Consider two partitions π and $\tilde{\pi}$ such that $\tilde{\pi} = \{A_1 \backslash B, B, A_2, \ldots, A_k\}$ and $\pi = \{A_1, A_2, \ldots, A_k\}$. Denote consumption bundles chosen under π and $\tilde{\pi}$ by x and $\tilde{x}$ respectively. If Assumption 1 holds, then*

1. $V_B(x) > V_{A_1\backslash B}(x) \Leftrightarrow V_B(\tilde{x}) > V_{A_1\backslash B}(\tilde{x}) \Leftrightarrow \tilde{x}_j < x_j, \tilde{x}_i > x_i$ *for any* $s_i \in A_1\backslash B$ *and* $s_j \in B$;

2. $V_B(x) < V_{A_1\backslash B}(x) \Leftrightarrow V_B(\tilde{x}) < V_{A_1\backslash B}(\tilde{x}) \Leftrightarrow \tilde{x}_j > x_j, \tilde{x}_i < x_i$ *for any* $s_i \in A_1\backslash B$ *and* $s_j \in B$;

3. $V_B(x) = V_{A_1\backslash B}(x) \Leftrightarrow V_B(\tilde{x}) = V_{A_1\backslash B}(\tilde{x}) \Leftrightarrow \tilde{x}_j = x_j, \tilde{x}_i = x_i$, $\tilde{x}_s = x_s$ *for any* $s_i \in A_1\backslash B$, $s_j \in B$, *and for all* $s \in A_k$ *and all* $k \neq 1$.

Moreover, if 1 or 2 from above hold, then one of the following is true:

(a) $\tilde{x}_s > x_s$ *for all* $s \in A_k$ *and all* $k \neq 1$;

(b) $\tilde{x}_s < x_s$ *for all* $s \in A_k$ *and all* $k \neq 1$;

(c) $\tilde{x}_s = x_s$ *for all* $s \in A_k$ *and all* $k \neq 1$.

The above theorem predicts the following. First, if a state does not belong to the affected event, then the consumption in this state will change in the same direction as consumption in other unaffected states and events. Second, if an event A gets split in two, $A\backslash B$ and B, and the current value of one of them is greater (e.g., $V_B(x) > V_{A\backslash B}(x)$), then event aversion will push the values toward each other, making consumption in event states with the higher value (B) decrease, and consumption in event states with the lower value ($A \backslash B$) increase. Third, if two events get aggregated, then the effect is opposite. The pressure that pushed the values closer due to event aversion is gone. Hence, the values at these events will pull away from each other. In other words, consumption in the states of the event with greater value will increase even more and the consumption in the states of the event with lower value will decrease.

Note: Unfortunately, Theorem 2 cannot be generalized to "one partition finer than another" context. The reason for that is any two affected events are outside events for one another. Hence, given that the direction of the effect on outside events cannot be predicted, we cannot forecast the impact that the affected events pose on one another.

The above theorems can be applied to the insurance market as the plans that consist of deductibles can be represented through Arrow assets. See the Appendix for more details. It is not unreasonable to imagine a situation in which an insurance company observes the current state aggregation, probabilities, state prices, and consumption. If the company is able to manipulate state aggregation, it will be interested in predicting corresponding changes in deductibles and/or consumption. If utilities were known, this would not be difficult, however, it is an unrealistic assumption. Given heterogeneity, even if the company observes consumer choices for 10 years, it would still be impossible to evaluate the utility function even if the preferences remained stable over time. The above theorem attempts to offer some predictions in this situation. Unfortunately, it relies on the comparison of current values at events ($V_B(x)$ vs. $V_{A \backslash B}(x)$), which still depend on the utility. However, in some situations, all possible utility functions that could generate the observable consumption imply the same relation between the values at the events. In this case, we are able to make predictions. Consider the following example.

Example 3: Suppose that the insurance company is interested in splitting event

$A = \{s_1, s_2, s_3, s_4\}$ into events $B_1 = \{s_1, s_4\}$ and $B_2 = \{s_2, s_3\}$. The probabilities are $P(s_1|B_1) = 0.3$ and $P(s_4|B_1) = 0.7$, while $P(s_2|B_2) = P(s_3|B_2) = 0.5$ and $P(B_1) = P(B_2)$. The current consumption is $x_1 = 2$, $x_2 = 4$, $x_3 = 7$, and $x_4 = 9$ and the state prices are $p_1 = 1.8$, $p_2 = 2.5$, $p_3 = 1.5$, and $p_4 = 1$. Note that by the Taylor expansion, we obtain the following.

$$u(x_2) = u(x_1) + u'(t_1)(x_2 - x_1), \text{ where } t_1 \in [x_1, x_2]$$
$$u(x_3) = u(x_2) + u'(t_2)(x_3 - x_2), \text{ where } t_2 \in [x_2, x_3]$$
$$u(x_4) = u(x_3) + u'(t_3)(x_4 - x_3), \text{ where } t_3 \in [x_3, x_4].$$

Hence, the values at the events can be written in terms of $u(x_1)$ and derivatives:

$$\begin{aligned} V_{B_1}(x) = 0.3u(x_1) + 0.7u(x_4) &= u(x_1) + 0.7u'(t_3)(x_4 - x_3) + 0.7u'(t_2)(x_3 - x_2) + 0.7u'(t_1)(x_2 - x_1) \\ &= u(x_1) + 1.4u'(t_3) + 2.1u'(t_2) + 1.4u'(t_1) \\ V_{B_2}(x) = 0.5u(x_2) + 0.5u(x_3) &= u(x_1) + u'(t_1)(x_2 - x_1) + 0.5u'(t_2)(x_3 - x_2) \\ &= u(x_1) + 2u'(t_1) + 1.5u'(t_2). \end{aligned}$$

Thus, $V_{B_1}(x) > V_{B_2}(x)$ if and only if $1.4u'(t_3) + 0.6u'(t_2) - 0.6u'(t_1) > 0$. Note that it depends only on the values of the utility derivatives for which we can establish bounds from the first-order conditions. Inside event A, the first-order condition implies $\frac{u'(x_k)}{u'(x_j)} = \frac{p_k}{p_j}\frac{P(s_j)}{P(s_k)}$. Hence, we get the following:

$$\frac{u'(x_1)}{u'(x_4)} = 4.2; \ \frac{u'(x_2)}{u'(x_4)} = 3.5; \ \frac{u'(x_3)}{u'(x_4)} = 2.1.$$

For simplicity, we normalize $u'(x_4) = 1$, then the bounds for the expression of interest are

$$\max(1.4u'(t_3) + 0.6u'(t_2) - 0.6u'(t_1)) = 2.94 > 0$$
$$\min(1.4u'(t_3) + 0.6u'(t_2) - 0.6u'(t_1)) = 0.14 > 0.$$

Thus, we conclude that $V_{B_1}(x) > V_{B_2}(x)$ and can predict that splitting event A into B_1 and B_2 will result in a decrease of consumption in states s_1 and s_4, and an increase of consumption in states s_2 and s_3. One can easily generalize this approach.

2.2.3 Portfolio Choice Example

Another interesting example is how portfolio choice can be affected by state aggregation. Consider an agent who would like to invest $1 into the financial market. There are three states of the world $\Omega = \{s_1, s_2, s_3\}$ that may occur with equal probabilities. Asset r is risky and it delivers 200% return in state s_3, while asset δ is a government bond that pays 20% in any state.

Table 1: Financial returns

	s_1	s_2	s_3
r	1	1	2
δ	1.2	1.2	1.2

For the purposes of the numerical example, we assume that $\phi(x) = 1-e^{-\rho x}$ if $\rho > 0$ and $u(x) = \ln x$. Note that in this case ρ has a natural interpretation of the coefficient of absolute event aversion.

The individual is going to invest proportion α into the risky asset and the rest into the bond. The question is what α will be chosen.

First, consider an agent who does not aggregate states, who then solves the following problem

$$\max_{\alpha} \frac{2}{3}\phi(u(1.2-0.2\alpha)) + \frac{1}{3}\phi(u(1.2+0.8\alpha)).$$

Second, consider an agent who aggregates states s_2 and s_3 into an event A. Her problem is

$$\max_{\alpha} \frac{1}{3}\phi(u(1.2-0.2\alpha)) + \frac{2}{3}\phi(0.5u(1.2-0.2\alpha)+0.5u(1.2+0.8\alpha)).$$

Table 2 shows the choices of α and the resulting consumption for different values of the absolute event aversion ρ.

Table 2: Choices for different values of ρ

ρ	α^{Ω}	$x_1^{\Omega} = x_2^{\Omega}$	x_3^{Ω}	α^{π}	$x_1^{\pi} = x_2^{\pi}$	x_3^{π}	$V_{s_1}^{\pi}(x)$	$V_A^{\pi}(x)$
0.1	0.90	1.02	1.92	0.97	1.01	1.98	0.01	0.34
0.5	0.63	1.07	1.70	0.87	1.03	1.90	0.03	0.33
1	0.46	1.11	1.57	0.76	1.05	1.81	0.05	0.32
5	0.14	1.17	1.31	0.37	1.13	1.50	0.12	0.26
10	0.08	1.18	1.26	0.22	1.16	1.37	0.15	0.23

Notice that as Theorem 1 predicts, the more event-averse the individual is, the closer the event values will be to each other (see $V_{s_1}^{\pi}(x)$ and $V_A^{\pi}(x)$), and, as a result, consumption will be in related states. In addition, partition Ω is obtained from π by splitting the event A. Hence, according to Theorem 2, consumption under state aggregation increases in the state with greater consumption (s_3), and it decreases further in the state with lower consumption (s_2).

Consequently, it is possible to imagine a situation in which the agent hires a fiduciary, who is interested in nudging the agent to invest more into the risky asset. By law, the fiduciary is not allowed to encourage investment in own interests directly, however, she is the person who formulates the frame (state aggregation) in which information is provided to the agent. Suppose that the agent's $\rho = 1$ and that she does not aggregate states. The fiduciary observes the agent's consumption and the fact that $x_2^\Omega = 1.11 < 1.57 = x_3^\Omega$. Based only on this information, the advisor knows that if s_2 and s_3 get aggregated, then consumption in s_3 will increase and consumption in s_2 will decrease. To increase consumption in s_3, the agent will have to buy more of the risky asset. Therefore, the financial adviser changes the frame in which information is presented to nudge the agent to state aggregate.

2.3 Ambiguity Paradoxes

In this section, we go over well-known thought experiments that demonstrate the problems that the most prominent theories have in explaining behavior. Dillenberger and Segal (2015) explain the paradoxes with the recursive non-expected utility model of Segal (1987). The model assumes that the agent has a distribution of priors over possible probability distributions and also uses a non-expected utility functional for evaluation of each possibility. SASEU offers an alternative explanation.

Ellsberg Paradox

Consider the following famous paradox from Ellsberg (1961): there is an urn that contains 90 colored balls – 30 balls are red and all other balls are black and yellow in unknown proportion. One ball is randomly picked from the urn and four lotteries are considered:

$$A_1: \text{ \$100 if the ball is red}$$

$$A_2: \text{ \$100 if the ball is black}$$

$$B_1: \text{ \$100 if the ball is red or yellow}$$

$$B_2: \text{ \$100 if the ball is black or yellow}$$

The subjects are offered to choose separately between A_1 and A_2, and between B_1

and B_2. It is a well-established fact that the majority of people prefer $A_1 \succ A_2$ and $B_2 \succ B_1$, which contradicts the classical SEU. Many models have been proposed to explain the paradox. We will add one more to the list:

The state space consists of the states red (R), black (B) and yellow (Y), i.e., $\Omega = \{R, B, Y\}$. Due to ambiguity, the agent might be naturally inclined to aggregate states B and Y into one event that we will call BY – thus, obtaining the state aggregation $\pi = \{R, BY\}$. The probabilities of the events in π are objective: $P(R) = \frac{1}{3}$ and $P(BY) = \frac{2}{3}$. The probabilities of the states in BY are subjective, however, there is no reason to believe that $P(B|BY) \neq P(Y|BY)$ due to the symmetry of the situation. Hence, $P(B|BY) = P(Y|BY) = 0.5$. Let us demonstrate how a SASEU-maximizer evaluates the above lotteries in this case:

$$V_\pi(A_1) = \frac{1}{3}\phi\left(u(100)\right) + \frac{2}{3}\phi\left(u(0)\right)$$
$$V_\pi(A_2) = \frac{1}{3}\phi\left(u(0)\right) + \frac{2}{3}\phi\left(0.5u(100) + 0.5u(0)\right).$$

Notice that $V_\pi(A_1) > V_\pi(A_2)$ if and only if

$$0.5\phi(u(100)) + 0.5\phi(u(0)) > \phi\left(0.5u(100) + 0.5u(0)\right). \tag{3}$$

The other two lotteries are evaluated as follows.

$$V_\pi(B_1) = \frac{1}{3}\phi\left(u(100)\right) + \frac{2}{3}\phi\left(0.5u(100) + 0.5u(0)\right)$$
$$V_\pi(B_2) = \frac{1}{3}\phi\left(u(0)\right) + \frac{2}{3}\phi\left(u(100)\right).$$

Moreover, $V_\pi(B_2) > V_\pi(B_1)$ if and only if Equation (3) holds. The condition is trivially satisfied if $\phi(\cdot)$ is concave.

Slightly-Bent Coin Problem

In Machina (2014), the author offers the following thought experiment: an agent needs to choose between two bets. The payout depends on a flip of a slightly-bent coin (it is unknown in which direction the coin is bent) and the color of the ball drawn from the urn. The urn contains two balls, each of them can be black or white. Hence, the state space consists of four states dependent on whether the ball is black or white and whether the coin lands heads or tails, i.e., $\Omega = \{BH, BT, WH, WT\}$. The bets are as follows.

$$\text{Bet I: } \$8,000 \text{ if } BH; \;\; -\$8,000 \text{ if } BT$$
$$\text{Bet II: } \$8,000 \text{ if } WT; \;\; -\$8,000 \text{ if } BT$$

Machina argues that the Choquet expected utility predicts indifference in this case, while real people might have strong preferences for one of the bets.

Consider a SASEU-maximizer that aggregates states with different balls and the same coin together. Thus, states BH and WH are aggregated into event H, and states BT and WT are aggregated into event T, forming a subjective partition $\pi = \{H, T\}$. First of all, there is no reason to believe that probabilities of white and black balls differ, so $P(W|H) = P(B|H) = P(W|T) = P(B|T) = 0.5$. In the same manner, there is no reason to believe that the coin is bent in a specific direction and the probabilities of heads and tails differ, implying $P(H) = P(T) = 0.5$. The value of each bet can be calculated as

$$V_\pi(I) = 0.5\phi\left(0.5u(8,000) + 0.5u(-8,000)\right) + 0.5\phi(u(0))$$
$$V_\pi(II) = 0.5\phi\left(0.5u(0) + 0.5u(-8,000)\right) + 0.5\phi\left(0.5u(0) + 0.5u(8,000)\right).$$

Note that as long as $\phi(\cdot)$ is not linear, the choice between Bet I and Bet II depends on the exact values of the functions $\phi(\cdot)$ and $u(\cdot)$. However, as long as $\phi(\cdot)$ is concave,

$II \succ I$.

Ambiguity at Low vs. High Outcomes Problem

This paradox was also proposed in Machina (2014). The subject is asked to choose between two urns. Both urns contain three balls, one of them is known to be red. Each of the other balls can be either black or white. The value of c is defined as certainty equivalent of a 50:50 bet for \$0 and \$100. The payoffs of the urns are shown in Table 3.

Table 3: Ambiguity at Low vs. High Outcomes

Urn	R	B	W
I	\$100	\$0	\$$c$
II	\$0	\$$c$	\$100

The most prominent ambiguity theories (MEU, Choquet, the smooth ambiguity, and variational preferences) predict indifference between the urns. However, Machina argues that the subjects might have a strong preference toward one of the urns.

An SASEU agent might naturally want to aggregate states B and W into event A for both urns. Then probabilities of the events R and A are objective: $P(R) = \frac{1}{3}$ and $P(A) = \frac{2}{3}$. In addition, given event A, there is no reason to believe that the probabilities of B and W are different for any of the urns, thus, $P(B|A) = P(W|A) = 0.5$. Hence, the value of each urn is as follows:

$$V_{\pi}(I) = \frac{1}{3}\phi\left(u(100)\right) + \frac{2}{3}\phi\left(0.5u(c) + 0.5u(0)\right) = \frac{1}{3}\phi\left(u(100)\right) + \frac{2}{3}\phi\left(0.25u(100) + 0.75u(0)\right)$$
$$V_{\pi}(II) = \frac{1}{3}\phi\left(u(0)\right) + \frac{2}{3}\phi\left(0.5u(c) + 0.5u(100)\right) = \frac{1}{3}\phi\left(u(0)\right) + \frac{2}{3}\phi\left(0.75u(100) + 0.25u(0)\right).$$

Notice that as long as $\phi(\cdot)$ is not linear, depending on $\phi(\cdot)$ and $u(\cdot)$, any behavior might be obtained. For example, if $\phi(x) = \sqrt{x}$, $u(0) = 0$ and $u(100) = 1$, then $I \succ II$.

50:51 Example

This paradox is described in Machina (2009). An urn contains 101 balls, 50 of which are marked with either 1 or 2, and 51 balls are marked with either 3 or 4. One ball is drawn at random. The subject is offered to choose between lotteries f_1 and f_2, and between f_3 and f_4, payoffs for which are shown in Table 4.

Table 4: 50:51 Example

Lottery	E_1	E_2	E_3	E_4
f_1	\$8,000	\$8,000	\$4,000	\$4,000
f_2	\$8,000	\$4,000	\$8,000	\$4,000
f_3	\$12,000	\$8,000	\$4,000	\$0
f_4	\$12,000	\$4,000	\$8,000	\$0

Lotteries f_3 and f_4 are obtained from f_1 and f_2 by shifting \$4,000 and adding that to \$8,000. Tail-separability in the Choquet model implies that $f_1 \succ f_2$ if and only if $f_3 \succ f_4$. Baillon et al. (2011) show that $f_1 \succ f_2$ implies $f_3 \succ f_4$ if preferences are represented by MEU, α-MEU, variational preferences, and the smooth ambiguity model with concave ϕ. Machina argues that there is no reason for some subjects not to show preference reversal in this case.

A SASEU-maximizer will aggregate states E_1 and E_2 into event E_{12} and states E_3 and E_4 into event E_{34}, hence, $\pi = \{E_{12}, E_{34}\}$. There is no reason for the probabilities of E_1 and E_2 given event E_{12} to be different, so $P(E_1|E_{12}) = P(E_2|E_{12}) = 0.5$. By analogy, $P(E_3|E_{34}) = P(E_4|E_{34}) = 0.5$. Probabilities of E_{12} and E_{34} are objective and equal to $\frac{50}{101}$ and $\frac{51}{101}$ respectively. Thus, the values of the lotteries are

$$V_\pi(f_1) = \frac{50}{101}\phi\left(u(8,000)\right) + \frac{51}{101}\phi\left(u(4,000)\right)$$
$$V_\pi(f_2) = \phi\left(0.5u(8,000) + 0.5u(4,000)\right)$$
$$V_\pi(f_3) = \frac{50}{101}\phi\left(0.5u(12,000) + 0.5u(8,000)\right) + \frac{51}{101}\phi\left(0.5u(4,000) + 0.5u(0)\right)$$
$$V_\pi(f_4) = \frac{50}{101}\phi\left(0.5u(12,000) + 0.5u(4,000)\right) + \frac{51}{101}\phi\left(0.5u(8,000) + 0.5u(0)\right).$$

Constructing a preference reversal example in this case is not as trivial as in other paradoxes, however, it is not impossible. Suppose that $u(0) = 0$, $u(4,000) = 0.5$, $u(8,000) = 1$, $u(12,000) = 2$ and $\phi(x) = \begin{cases} x, & \text{if } x \leq 1 \\ x^2, & \text{if } x > 1 \end{cases}$. Then we obtain that $f_2 \succ f_1$ and $f_3 \succ f_4$. However, note that if $\phi(\cdot)$ is concave, then only $f_2 \succ f_1$ and $f_4 \succ f_3$ is possible.

Reflection Example

The following thought experiment was also introduced in Machina (2009). An urn contains 100 balls, half of which are marked with either 1 or 2, and the other half are marked with either 3 or 4. One ball is drawn at random. The subject is offered to choose between lotteries f_5 and f_6, and between f_7 and f_8, payoffs for which are shown in Table 5.

Table 5: Reflection Example

Lottery	E_1	E_2	E_3	E_4
f_5	\$4,000	\$8,000	\$4,000	0
f_6	\$4,000	\$4,000	\$8,000	0
f_7	0	\$8,000	\$4,000	\$4,000
f_8	0	\$4,000	\$8,000	\$4,000

The Choquet model implies that $f_5 \succ f_6$ if and only if $f_7 \succ f_8$, because f_7 and f_8 are obtained from f_5 and f_6 by switching 0 and \$4,000. Machina argues that there is no difference between lotteries f_5 and f_8, and between f_6 and f_7, implying that $f_5 \succ f_6$ if and only if $f_8 \succ f_7$. Thus, only indifference between four lotteries would be possible. In an experimental study, L'Haridon and Placido (2010) found that $f_6 \succ f_5$ and $f_7 \succ f_8$. As a result, the Choquet model is rejected. In addition, Baillon et al. (2011) demonstrate that MEU, variational preferences, and the smooth ambiguity model do not allow $f_6 \succ f_5$ and $f_7 \succ f_8$. The authors also show that the preferences might be represented by α-MEU, however, it would require unreasonable priors.

An SASEU-maximizer will aggregate states E_1 and E_2 into event E_{12} and states E_3 and E_4 into event E_{34}. There is no reason for the probabilities of E_1 and E_2 given event E_{12} to be different, so $P(E_1|E_{12}) = P(E_2|E_{12}) = 0.5$. By analogy, $P(E_3|E_{34}) = P(E_4|E_{34}) = 0.5$. The probabilities of E_{12} and E_{34} are objective and equal 0.5. Thus, the values of the lotteries are

$$V_\pi(f_5) = V_\pi(f_8) = 0.5\phi\left(0.5u(4,000) + 0.5u(8,000)\right) + 0.5\phi\left(0.5u(4,000) + 0.5u(0)\right)$$
$$V_\pi(f_6) = V_\pi(f_7) = 0.5\phi\left(u(4,000)\right) + 0.5\phi\left(0.5u(8,000) + 0.5u(0)\right).$$

Notice that a concave $\phi(\cdot)$ implies that $f_6 \succ f_5$ and $f_7 \succ f_8$.

3 Identification in the Market

In this section, we show how to identify state aggregation from choices of Arrow securities[2] and their prices. Even if Arrow assets are not directly available in the market, as long as the market is complete, Arrow prices can always be uniquely recovered.

3.1 Non-Parametric Identification Example

Suppose that $\Omega = \{s_1, s_2, s_3\}$, partition $\pi = \{A_1, A_2\}$, where $A_1 = \{s_1\}$ and $A_2 = \{s_2, s_3\}$, with some priors p_{A_1}, p_{A_2}, $P(s_1|A_1) = 1$, $P(s_2|A_2)$ and $P(s_3|A_2)$. Consider that a complete set of Arrow securities is available on the market. The agent purchases a bundle of Arrow assets that maximizes her value given a certain income I and the price p_i of an Arrow security that pays 1 in state i:

$$\max_x V_\pi(x_1, x_2, x_3)$$
$$\text{s.t. } p_1 x_1 + p_2 x_2 + p_3 x_3 = I.$$

[2] An Arrow security pays one unit in a specified state and zero otherwise.

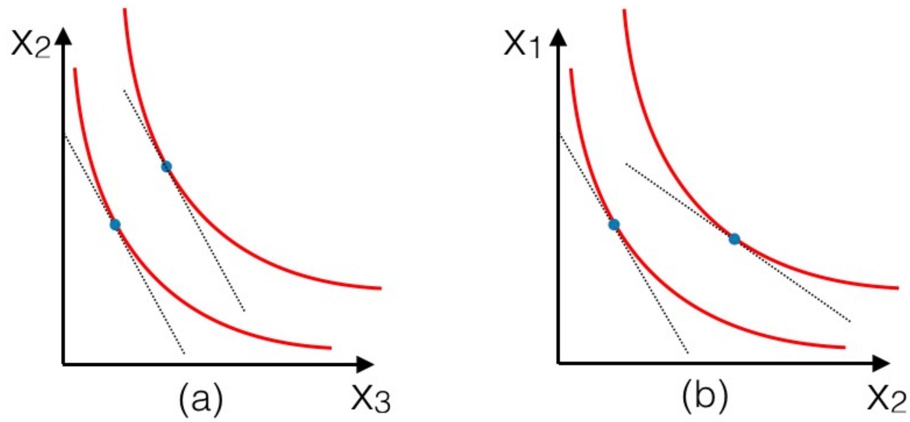


Figure 1: MRS

Choices $x = (x_1, x_2, x_3)$, prices $p = (p_1, p_2, p_3)$ and income I are observed. We assume that all possible combinations of (p, I) are available. The purpose is to identify the state aggregation π, utility function $u(\cdot)$, transformation function $\phi(\cdot)$, and priors p_{A_1}, p_{A_2}, $P(s_2|A_2)$, $P(s_3|A_2)$.

The intuition behind identification of state aggregation is as follows. The states that belong to the same event will have stable MRS, i.e., it will depend only on corresponding prices and probabilities. Hence, for all prices p_2 and p_3 such that $\frac{p_2}{p_3}\frac{P(s_3)}{P(s_2)}$ is constant, MRS for consumption in states s_2 and s_3 must be the same (see Figure 1(a)).

On the other hand, if states belong to different events, then MRS also depends on consumption in other states. This conclusion implies that for all prices p_1 and p_2 such that $\frac{p_1}{p_2}\frac{P(s_2)}{P(s_1)}$ is constant, MRS for consumption in states s_1 and s_2 differs (see Figure 1(b)).

Note: MRS of an SEU-maximizer between any two states would always be stable like in Figure 1(a).

Now we will formally show the identification procedure in the proposed example.

The agent solves her maximization problem and chooses some bundle x:

$$\max_x p_{A_1}\phi(u(x_1)) + p_{A_2}\phi\left(P(s_2|A_2)u(x_2) + P(s_3|A_2)u(x_3)\right)$$
$$\text{s.t. } p_1^x x_1 + p_2^x x_2 + p_3^x x_3 = I.$$

Hence, if λ is a Lagrange multiplier, then the first order conditions are

$$p_{A_1}\phi'(u(x_1))u'(x_1) = \lambda p_1^x$$
$$p_{A_2}\phi'\left(P(s_2|A_2)u(x_2) + P(s_3|A_2)u(x_3)\right)p(s_2|A_2)u'(x_2) = \lambda p_2^x$$
$$p_{A_2}\phi'\left(P(s_2|A_2)u(x_2) + P(s_3|A_2)u(x_3)\right)p(s_3|A_2)u'(x_3) = \lambda p_3^x.$$

Two states from the same event: If we pick s_2 and s_3, we obtain

$$\frac{p(s_2|A_2)}{p(s_3|A_2)}\frac{u'(x_2)}{u'(x_3)} = \frac{p_2^x}{p_3^x}. \tag{4}$$

Now choose some other bundle $y = (y_1, y_2, y_3)$ such that $y_2 = x_2$, but $y_3 \neq x_3$. Then we observe similar first-order conditions:

$$\frac{p(s_2|A_2)}{p(s_3|A_2)}\frac{u'(y_2)}{u'(y_3)} = \frac{p_2^y}{p_3^y}. \tag{5}$$

After dividing Equation (4) by Equation (5), we obtain

$$\frac{u'(y_3)}{u'(x_3)} = \frac{p_2^x}{p_3^x}\frac{p_3^y}{p_2^y}. \tag{6}$$

Two states from different events: We repeat the above derivation for states s_1 and s_2:

$$\frac{p_{A_1}}{p_{A_2}p(s_2|A_2)}\frac{\phi'(u(x_1))u'(x_1)}{\phi'\left(P(s_2|A_2)u(x_2) + P(s_3|A_2)u(x_3)\right)u'(x_2)} = \frac{p_1^x}{p_2^x}. \tag{7}$$

Choose another bundle $z = (z_1, z_2, z_3)$ such that $z_1 = x_1$, but $z_2 \neq x_2$. After dividing Equation (7) by the corresponding first-order condition for bundle z, we get

$$\frac{\phi'\left(P(s_2|A_2)u(z_2)+P(s_3|A_2)u(z_3)\right)}{\phi'\left(P(s_2|A_2)u(x_2)+P(s_3|A_2)u(x_3)\right)}\frac{u'(z_2)}{u'(x_2)}=\frac{p_1^x}{p_2^x}\frac{p_2^z}{p_1^z}. \tag{8}$$

Compare now Equation (6) and Equation (8). In Equation (6), the left side does not depend on the values of x_1, y_1, or $x_2 = y_2$. Thus, if we pick other bundles with the same x_3 and y_3, we will obtain the same value of the ratio in Equation (6). In Equation (8), the left side depends on x_3 and z_3. So as long as $\phi(\cdot)$ is not a linear function, changing x_3 and z_3 while keeping everything else constant will result in different values on the right side. Hence, we know that the state aggregation is $\pi = \{\{s_1\}, \{s_2, s_3\}\}$.

Also note that by choosing different values of x_3 and y_3 in Equation (6), we can identify $u(\cdot)$ up to affine transformation. After that we consider Equation (4) again and identify the probability ratio $\frac{P(s_2|A_2)}{P(s_3|A_2)}$. This means we can identify the probabilities themselves:

$$\frac{P(s_2|A_2)}{P(s_3|A_2)}=c \text{ and } P(s_2|A_2)+P(s_3|A_2)=1 \Rightarrow P(s_2|A_2)=\frac{c}{c+1} \text{ and } P(s_3|A_2)=\frac{1}{c+1}.$$

To identify $\phi(\cdot)$, we consider Equation (8) again: $u(\cdot)$, $P(s_2|A_2)$ and $P(s_3|A_2)$ have already been identified, hence, we can obtain the value of the ratio $\frac{\phi'(P(s_2|A_2)u(z_2)+P(s_3|A_2)u(z_3)))}{\phi'(P(s_2|A_2)u(x_2)+P(s_3|A_2)u(x_3)))}$. By choosing different values of x_2, x_3, z_2 and z_3, we identify $\phi(\cdot)$ up to affine transformation.

The only unknown variables left are p_{A_1} and p_{A_2}. However, we obtain $\frac{p_{A_1}}{p_{A_2}}$ from Equation (7). Finally, given that $p_{A_1} + p_{A_2} = 1$, we identify p_{A_1} and p_{A_2}.

3.2 Identification

The agent wants to buy a portfolio of securities and aggregates states of the world into some partition. The purpose of this section is to identify the agent's state aggregation and priors from choices of Arrow assets and their prices. To do so, we generalize the non-parametric example from Section 3.1; however, the idea behind the method stays the same.

Assumption 2 forbids aggregation of all states together. It is necessary because the agent's behavior under no aggregation and under full aggregation would follow SEU, making it impossible to identify the exact situation without any additional information[3]. The agent that aggregates the whole space shows less risk aversion than someone who does not aggregate at all. However, this observation would not be enough to distinguish between such individuals.

Assumption 2. *π is not equal to $\{\Omega\}$.*

Theorem 3. *If Assumptions 1 and 2 hold, then the state aggregation π and probabilities p_A and $P(s|A)$ are identified for any $s \in A \in \pi$. In addition, functions $u(\cdot)$ and $\phi(\cdot)$ are identified up to affine transformation.*

The proof of the theorem is provided in the Appendix and it follows the identification procedure shown in the previous subsection.

Note: The idea of state aggregation and the above theorem can be easily generalized to a finite sequence of finer partitions. The identification procedure would be the same, however, it would require several rounds. See the Appendix for more detail.

4 Axiomatization

4.1 Preliminaries

Suppose that X is a compact convex subset of consequences in $\mathbb{R}$, and Ω is a finite set of states of the world with an algebra Σ of subsets of Ω. We denote $\mathcal{F}$ as a set of all acts, Σ-measurable finite step functions: $\Omega \to \Delta X$. Let $\mathcal{M}$ be a set of elements π, such that $\pi \subset \Sigma$ is a partition of Ω. Partitions of the state space represent different ways of state aggregation.

[3] As long as $\phi(\cdot)$ is not linear, Ω and $\{\Omega\}$ will produce different choices. Both will be consistent with SEU, however, with different utilities.

We denote for all $x, y \in \mathcal{F}, A \in \Sigma$, xAy an act: $xAy(s) = x(s)$ if $s \in A$, and $xAy(s) = y(s)$ if $s \notin A$. The mixture of acts is defined statewise. We will abuse notation and define X as a set of constant acts in what follows below.

4.2 Axioms and Representation

The purpose of this section is to provide axioms of preference relation $\succeq$ between acts x over $\mathcal{F}$ that can be represented by the model of state aggregation. We take as a primitive a preference relation between acts. After that, we assume an exogenous partition of Ω and induce conditional preferences whenever we can guarantee that they are well-defined.

Axiom 1. *$\succeq$ is complete, transitive and not degenerate.*

Axiom 2. *For all $x, y, h \in \mathcal{F}$, if $x \succ y$, and $y \succ h$, then there exist $\lambda, \mu \in (0,1)$ such that $\lambda x + (1-\lambda)h \succ y$ and $g \succ \mu x + (1-\mu)h$.*

Axiom 3. *For all $x, y \in \mathcal{F}$, if $x(s) \succeq y(s)$ for all $s \in \Omega$, then $x \succeq y$.*

Axiom 4. *For all $x, y, h \in \mathcal{F}$, if $\lambda \in (0,1]$: $x \succeq y \Leftrightarrow \lambda x + (1-\lambda)h \succeq \lambda y + (1-\lambda)h$.*

Definition 3. *$\succeq$ is said to exhibit SEU representation if and only if there exist probabilities $P(s)$ for any state $s \in \Omega$, and a continuous monotone function $u : X \to \mathbb{R}$ such that*

$$x \succeq y \Leftrightarrow \sum_{s\in\Omega} P(s)u(x(s)) \geq \sum_{s\in\Omega} P(s)u(y(s)).$$

Axioms 1–4 is the standard set of axioms that delivers SEU representation. Axiom 4 is the well-known independence axiom that we have to replace with something weaker in order to obtain SASEU representation.

For everything that follows, consider that partition π is the exogenously given state aggregation.

Axiom 5 (π-restricted sure thing principle)**.** *For all events $A \in \pi$ and all $x, y, h, h' \in \mathcal{F}$, $xAh \succeq yAh \Leftrightarrow xAh' \succeq yAh'$.*

The independence axiom is an extension of the original Savage's Sure Thing Principle that would require $xAh \succeq yAh \Leftrightarrow xAh' \succeq yAh'$ for all $A \in \Sigma$. We restrict it from the entire algebra Σ to partition π to guarantee that the value of the act at the event is the only aspect that matters in act evaluation, and not each state value separately. However, this applies only in the given partition.

Now, we are ready to define conditional preferences $\succeq_A$ for all events $A \in \pi$:

Definition 4. *For all $x, y, h \in \mathcal{F}$: $x \succeq_A y \Leftrightarrow xAh \succeq yAh$.*

Lemma 1. *If Axioms 1-3 and 5 hold, then $\succeq_A$ are complete, transitive, monotone and continuous for all $A \in \pi$.*

To obtain the SASEU representation, that implies SEU-like functionals on each event and over events, we need to add the independence axioms for conditional and ex-ante preferences.

Axiom 6 (Conditional Independence)**.** *For all events $A \in \pi$, all acts $x, y, h \in \mathcal{F}$ and any $\alpha \in (0,1)$: $x \succeq_A y$ if and only if $\alpha x + (1-\alpha)h \succeq_A \alpha y + (1-\alpha)h$.*

Definition 5. *Suppose that $\pi = \{A_1, A_2, \ldots, A_n\}$, then for any act $h \in \mathcal{F}$ and event $A_i \in \pi$ define constant acts $x_h^{A_i} \in X$ and $h^\pi \in \mathcal{F}$: $x_h^{A_i} \sim_{A_i} h$ and $h^\pi \sim x_h^{A_1} A_1 x_h^{A_2} A_2 \ldots x_h^{A_n}$.*

Axiom 7 (Ex-ante Independence)**.** *For any acts $x, y, h \in \mathcal{F}$ and any $\alpha \in (0,1)$: $x \succeq y$ if and only if $\alpha x^\pi + (1-\alpha)h^\pi \succeq \alpha y^\pi + (1-\alpha)h^\pi$.*

Definition 6. *$\succeq$ is said to exhibit SASEU representation if there exist a partition of the state space π, probabilities $P(s|A)$ and $P(A|\pi)$ for any state $s \in A \in \pi$, a continuous monotone function $u : X \to \mathbb{R}$, and an increasing function $\phi : \mathbb{R} \to \mathbb{R}$*

such that

$$x \succeq y \Leftrightarrow \sum_{A\in\pi} P(A)\phi\left(\sum_{s\in A} P(s|A)u(x(s))\right) \geq \sum_{A\in\pi} P(A)\phi\left(\sum_{s\in A} P(s|A)u(y(s))\right).$$

Theorem 4. *$\succeq$ exhibits SASEU representation with (π, u, ϕ, P) if and only if Axioms 1-3 and 5-7 hold. Moreover,*

1. *$\succeq$ can be represented by SASEU with (π, u, ϕ, P) and with $(\pi' \neq \pi, u', \phi', P')$ if and only if $u(\cdot)$ is an affine transformation of $u'(\cdot)$, and both $\phi(\cdot)$ and $\phi'(\cdot)$ are linear;*

2. *$\succeq$ can be represented by SASEU with (π, u, ϕ, P) and with (π, u', ϕ', P'), where ϕ and ϕ' are not linear, if and only if $P = P'$, $u(\cdot)$ and $\phi(\cdot)$ are some affine transformations of $u'(\cdot)$ and $\phi'(\cdot)$.*

In addition to establishing the representation, Theorem 4 states the requirements for its uniqueness. Note that the same preferences can be represented by SASEU with two different state aggregations π and π' if and only if $\phi(\cdot)$ is linear, i.e., the agent is an SEU-maximizer. Hence, as long as $\phi(\cdot)$ is not linear, state aggregation and subjective priors are unique, while utility $u(\cdot)$ and $\phi(\cdot)$ are unique up to an affine transformation.

Note that the model can be easily generalized and the SEU functional at each evaluation stage can be substituted with some other functional. The general characterization of the model relies on Axioms 1-3, 5 and the certainty independence[4]. Theorem 5 in Section A.2 of the Appendix proposes the general representation of the preferences.

[4]See Axiom 8 in Section A.2 of the Appendix.

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A Appendix

A.1 Deductibles as Arrow Assets

Note that the problem of choosing an insurance plan that contains deductibles and is priced linearly can be represented as a choice among Arrow securities. To see this, consider the following example.

Example: Suppose that an agent needs to choose an insurance plan while considering three states of the world: robbery with losses L_1; a natural disaster with losses L_2; and no accident, which implies no loss. The price of insurance consists of the sum of prices of chosen deductibles in both states of occurrence. We assume that price $q(d)$ of each deductible d is linear in its amount d: $q(d) = a - p_d d$, where a is a constant state premium. Thus, we can define the price of a \$1 deductible decrease as $p_d = q(d) - q(d+1)$, which is constant for any amount d due to assumed linearity. Notice that p_d is the price of a corresponding Arrow security: decreasing the deductible by \$1 in some state is equivalent to increasing consumption by \$1 in the same state.

We denote the deductible in the case of robbery as d_1, the price of \$1 of deductible in this state as p_{d_1}, and a_1 as the state premium. Similarly, d_2, p_{d_2}, and a_2 are the deductible, the \$1 price, and the state premium in the case of the natural disaster, respectively. Thus, the total price of the insurance plan (d_1, d_2) is $p = (L_1 - d_1)p_{d_1} + a_1 + (L_2 - d_2)p_{d_2} + a_2$. The agent has income I and optimizes consumption $x = (x_1, x_2, x_3)$ in all states of the world given some state aggregation π. All the money left (s) after paying for insurance is used for consumption during the year. Thus, the agent's problem is

$$\max_x V_\pi(x_1, x_2, x_3)$$

$$\text{s.t. } (L_1 - d_1)p_{d_1} + (L_2 - d_2)p_{d_2} + a_1 + a_2 + s = I$$

$$x_1 = s - d_1;\ x_2 = s - d_2;\ x_3 = s.$$

We now rewrite the problem in terms of Arrow securities. Notice that the consumption bundle $x = (x_1, x_2, x_3)$ is exactly the portfolio of Arrow assets that the agent chooses. The only thing left to do is to find Arrow prices (p_1, p_2, p_3) and rewrite the budget constraint in an appropriate form. Denote $\tilde{I} = I - L_1 p_1 - L_2 p_2 - a_1 - a_2$ the agent's total endowment when taking potential losses and all state premiums into account. Also, as mentioned above, $p_1 = p_{d_1}$ and $p_2 = p_{d_2}$. Then, we can rewrite the budget constraint as

$$-d_1 p_{d_1} - d_2 p_{d_2} + s = (x_1 - s)p_1 + (x_2 - s)p_2 + s = x_1 p_1 + x_2 p_2 + x_3(1 - p_1 - p_2) = \tilde{I}.$$

Thus, $p_3 = 1 - p_1 - p_2$ and the agent's problem is

$$\max_x V_\pi(x_1, x_2, x_3)$$

$$\text{s.t. } x_1 p_1 + x_2 p_2 + x_3 p_3 = \tilde{I}.$$

As a consequence, if we observe the choices of deductibles with their prices and the agent's income, we can recover the amount of Arrow securities/consumption in each state.

A.2 Axiomatization

Proof of Lemma 1. Completeness of $\succeq_A$ follows directly from the π-restricted sure thing principle.

Transitivity: Suppose that $f \succeq_A g$ and $g \succeq_A e$, then $fAh \succeq gAh$ and $gAh \succeq eAh$ for any $h \in \mathcal{F}$, which due to transitivity of $\succeq$ implies that $fAh \succeq eAh$ for any $h \in \mathcal{F}$. Hence, $f \succeq_A e$.

Monotonicity: Suppose that $f(s) \succeq g(s)$ for all $s \in A$, then by monotonicity of $\succeq$: $fAh \succeq gAh$ for all $h \in \mathcal{F}$. Hence, $f \succeq_A g$.

Continuity: Suppose that $f \succ_A g \succ_A e$, then for any $h \in \mathcal{F}$: $fAh \succ gAh \succ eAh$. By Axiom 2, there exist $\lambda_h, \mu_h \in (0,1)$ such that $\lambda_h fAh + (1-\lambda_h)eAh \succ gAh$ and $gAh \succ \mu_h fAh + (1-\mu_h)eAh$. Thus, $(\lambda_h f + (1-\lambda_h)e)Ah \succ gAh \succ (\mu_h f + (1-\mu_h)e)Ah$. Take $\lambda = \max_{h\in\mathcal{F}} \lambda_h$ and $\mu = \min_{h\in\mathcal{F}} \mu_h$, then $(\lambda f + (1-\lambda)e)Ah \succ gAh \succ (\mu f + (1-\mu)e)Ah$ for any $h \in \mathcal{F}$, hence, $\lambda f + (1-\lambda)e \succ_A g \succ_A \mu f + (1-\mu)e$. □

Proof of Theorem 4.

Axiom 8. *For all constant acts $x, y, z \in X$ and $\lambda \in (0,1)$: $x \succeq y$ if and only if $\lambda x + (1-\lambda)z \succeq \lambda y + (1-\lambda)z$.*

Theorem 5. *A binary relation $\succeq$ satisfies Axioms 1-3, 5 and 8 if and only if there exist a unique up to affine transformation nonconstant continuous monotone function $u : X \to \mathbb{R}$, monotone continuous functionals $I : \mathbb{R}^{|\pi|} \to \mathbb{R}$ and $I_{A_i} : \mathbb{R}^{|A_i|} \to \mathbb{R}$ for each $A_i \in \pi$ such that $\succeq_A$ is represented by the unique preference functional $V_A(\cdot) : \mathcal{F} \to \mathbb{R}$, and $\succeq$ is represented by unique $V_\pi(\cdot) : \mathcal{F} \to \mathbb{R}$ such that*

$$V_\pi(f) = I(V_{A_1}(f), V_{A_2}(f), \ldots, V_{A_n}(f))$$
$$V_{A_i}(f) = I_{A_i}(u(f)).$$

Proof. **Sufficiency:** $\succeq$ is continuous and independent preference relation on a mixture space X, thus, by the Mixture Space Theorem, it can be represented by a continuous and monotone utility function $u : X \to \mathbb{R}$. Note that the same applies to $\succeq_A$ on X.

Let $\succeq_A^*$ be preference relation on $u(X)$: $u(f) \succeq_A^* u(g)$ if and only if $f \succeq_A g$. $\succeq_A^*$ is continuous, independent and monotone preference relation on $u(X)$, hence, there exists a continuous and monotone $I_A : u(X) \to \mathbb{R}$ that represents $\succeq_A^*$. Thus, define $V_A(f) = I_A(u(f))$.

Let $\succeq_\pi$ be preference relation such that

$$V_{A_1}(f)A_1V_{A_2}(f)A_2...V_{A_n}(f) \succeq_\pi V_{A_1}(g)A_1V_{A_2}(g)A_2...V_{A_n}(g) \Leftrightarrow f \succeq g.$$

$\succeq_\pi$ is continuous, independent and monotone preference relation, hence, there exists continuous and monotone $I : \mathbb{R}^{|\pi|} \to \mathbb{R}$ that represents $\succeq_\pi$. Define $V_\pi(f) = I(V_{A_1}(f), \ldots, V_{A_n}(f))$.

Necessity: Define $f \succeq g$ if and only if $V_\pi(f) \geq V_\pi(g)$. Axiom 1 is straightforward. For Axiom 2, we want to show that the π-restricted sure thing principle holds any event $A \in \pi$. Thus, if $fAh \succeq gAh$ for some f, g and h in $\mathcal{F}$, then $fAh' \succeq gAh'$ for any $h' \in \mathcal{F}$. Without loss of generality, suppose we demonstrate the property for A_1:

$$fA_1h \succeq gA_1h \Leftrightarrow V_\pi(fA_1h) \geq V_\pi(gA_1h)$$

$$\Leftrightarrow I(V_{A_1}(f), \dots, V_{A_n}(h)) \geq I(V_{A_1}(g), \dots, V_{A_n}(h))$$

$$\text{by monotonicity } \Leftrightarrow V_{A_1}(f) \geq V_{A_1}(g)$$

$$\text{by monotonicity again} \Leftrightarrow I(V_{A_1}(f), \dots, V_{A_n}(h')) \geq I(V_{A_1}(g), \dots, V_{A_n}(h'))$$

$$\Leftrightarrow V_\pi(fA_1h') \geq V_\pi(gA_1h') \Leftrightarrow fA_1h' \succeq gA_1h'.$$

□

SASEU Representation: Axiom 8 follows from Axioms 6 and 7. Theorem 5 establishes the existence of the functionals $V_\pi(\cdot)$ and $V_A(\cdot)$ for all $A \in \pi$. Axioms 6 and 7 are standard independence axioms for corresponding preferences, hence, the SASEU representation follows. Now we need a couple of Lemmas in order to establish the uniqueness result.

Suppose that there are two different partitions, π and π'. Denote a set $\tilde{\pi} = \{C \in \Sigma : \exists A \in \pi, B \in \pi' : C = A \cap B\}$. Also, denote $\mathcal{A}$ a set of all events A such that $\forall f, g, h, h' \in \mathcal{F}$: $fAh \succ gAh \Leftrightarrow fAh' \succ gAh'$. Note that all events in π and π' satisfy this property due to Axiom 5.

Lemma 2. *If $\pi, \pi' \in \mathcal{A}$ and $C \in \tilde{\pi}$, then $C \in \mathcal{A}$.*

Proof. Take events $A \in \pi$ and $B \in \pi'$ such that $C = A \cap B$. Then, $fCx \succeq_A gCx \Leftrightarrow fCx(A \setminus C)h \succeq gCx(A \setminus C)h$. The last relation is equivalent to $fCh(B \setminus C)x(A \setminus C)h \succeq gCh(B \setminus C)x(A \setminus C)h \Leftrightarrow fCh \succeq_B gCh$.

Now, by providing the same argument from event B back to A, one can easily obtain that $fCh \succeq_A gCh$. □

Lemma 3. *Two different SASEU representations of $\succeq$ that satisfy Axioms 1–3 and 5–7 with partitions π and π' exist if there exists a SASEU representation with a denser partition $\tilde{\pi} = \{C : A \in \pi, B \in \pi', A \cap B = C\}$.*

Proof. First, note that $\tilde{\pi} = \{C : A \in \pi, B \in \pi', A \cap B = C\} \subseteq \mathcal{A}$. Now we just need to show that Axioms 6 and 7 hold for this partition too.

Note that for any $C \in \tilde{\pi}$ we can define conditional preferences $\succeq_C$ because $C \subseteq \mathcal{A}$. Axiom 6 for $\succeq_C$ follows trivially from Axiom 6 for $\succeq_A$, where $C \subseteq A \in \pi$.

Now we are left to show ex-ante independence for $\tilde{\pi}$. Any event $A \in \pi$ consists of a number of events from $\tilde{\pi}$: $A = C_1 \cup C_2 \cup \cdots \cup C_k$. Then for any act $f \in \mathcal{F}$: $f \sim_A x_f^A \sim_A x_f^{C_1} C_1 x_f^{C_2} C_2 \ldots x_f^{C_k}$. By conditional independence:

$$\alpha x_f^A + (1-\alpha) x_h^A \sim_A \alpha x_f^{C_1} C_1 x_f^{C_2} C_2 \ldots x_f^{C_k} + (1-\alpha) x_h^{C_1} C_1 x_h^{C_2} C_2 \ldots x_h^{C_k}.$$

Thus, $f \succeq g$ if and only if $\alpha f^\pi + (1-\alpha) h^\pi \succeq g^\pi + (1-\alpha) h^\pi$ if and only if

$$\alpha x_f^{A_1} A_1 x_f^{A_2} A_2 \ldots x_f^{A_n} + (1-\alpha) x_h^{A_1} A_1 x_h^{A_2} A_2 \ldots x_h^{A_n} \succeq \alpha x_g^{A_1} A_1 x_g^{A_2} A_2 \ldots x_g^{A_n} + (1-\alpha) x_h^{A_1} A_1 x_h^{A_2} A_2 \ldots x_h^{A_n}$$

if and only if

$$\alpha x_f^{C_1} C_1 x_f^{C_2} C_2 \ldots x_f^{C_t} + (1-\alpha) x_h^{C_1} C_1 x_h^{C_2} C_2 \ldots x_h^{C_t} \succeq \alpha x_g^{C_1} C_1 x_g^{C_2} C_2 \ldots x_g^{C_t} + (1-\alpha) x_h^{C_1} C_1 x_h^{C_2} C_2 \ldots x_h^{C_t},$$

which is by definition $f^{\tilde{\pi}} \succeq g^{\tilde{\pi}}$. □

Lemma 4. *If $C \in \mathcal{A}$ and $C \subseteq A \in \pi$, then for any act $f \in \mathcal{F}$ and $x_f^C \in X$:*

$$u(x_f^C) = \sum_{s \in C} P(s|C) u(f(s)),$$

where $P(s|C) = \frac{P(s|A)}{\sum_{s \in C} P(s|A)}$.

Proof. Note that $fCh \sim_A x_f^C Ch$ for any $f, h \in \mathcal{F}$, then

$$V_A(fCh) = \phi\left(\sum_{s \in C} P(s|A) u(f(s)) + \sum_{s \in A \setminus C} P(s|A) u(h(s))\right)$$
$$V_A(x_f^C Ch) = \phi\left(\sum_{s \in C} P(s|A) u(x_f^C) + \sum_{s \in A \setminus C} P(s|A) u(h(s))\right).$$

$V_A(fCh) = V_A(x_f^C Ch)$ implies that

$$\sum_{s\in C} P(s|A)u(f(s)) = \sum_{s\in C} P(s|A)u(x_f^C) \Rightarrow$$
$$u(x_f^C) = \sum_{s\in C} \frac{P(s|A)}{\sum_{s\in C} P(s|A)} u(f(s)).$$

□

Now we can demonstrate the uniqueness. The sufficiency part is trivial. Thus, we will show only necessity.

By Lemma 2 and Lemma 3 there exists a $\tilde{\pi}$ that also represents $\succeq$. Utility $u(\cdot)$ is defined up to affine transformation and by Lemma 4,

$$u(x_f^C) = \sum_{s\in C} \frac{P(s|A)}{\sum_{s\in C} P(s|A)} u(f(s)).$$

We also know that $\tilde{u}(x_f^C) = \sum_{s\in C} \tilde{P}(s|C)\tilde{u}(f(s))$ and $u(\cdot)$ and $\tilde{u}(\cdot)$ must agree on constant acts. Hence, one is an affine transformation of another. The same argument applies to π'.

Moreover, for $A = C_1 \cup \cdots \cup C_k$ that consists of more than one event from $\tilde{\pi}$,

$$\phi\left(\sum_{s\in A} P(s|A)u(f(s))\right) = \sum_{C_i\in A} \tilde{P}(C_i|A)\tilde{\phi}\left(\sum_{s\in C_i} \frac{P(s|A)}{\sum_{s\in C_i} P(s|A)} u(f(s))\right).$$

The latter implies that $\phi(\cdot)$ and $\tilde{\phi}(\cdot)$ are linear and affine transformations of each other.

□

A.3 Comparative Statics

A.3.1 Proof of Theorem 1

First of all, note that the definition of ϕ being more concave than $\tilde{\phi}$ implies that for all $x, y \in \mathbb{R}$ such that $x < y$:

$$\frac{\phi'(x)}{\phi'(y)} > \frac{\tilde{\phi}'(x)}{\tilde{\phi}'(y)} > 1.$$

For any events $A, B \in \pi$ and states $s_i \in A$ and $s_j \in B$ we will have the following condition:

$$\frac{\phi'(V_A(x))}{\phi'(V_B(x))} \frac{u'(x_i)}{u'(x_j)} = \frac{\tilde{\phi}'(V_A(\tilde{x}))}{\tilde{\phi}'(V_B(\tilde{\tilde{x}}))} \frac{u'(\tilde{x}_i)}{u'(\tilde{x}_j)}. \tag{9}$$

Lemma 5. *Suppose $\phi(\cdot)$ is more concave than $\tilde{\phi}(\cdot)$, keeping income I, $u(\cdot)$, π, probabilities $P(s)$ and prices p_s $\forall s \in \Omega$ the same for both consumers. If Assumption 1 holds, then*

1. *$x_s > \tilde{x_s}$ for some $s \in A \in \pi \Leftrightarrow x_s > \tilde{x_s}$ for all $s \in A \in \pi \Leftrightarrow V_A(x) > V_A(\tilde{x})$;*

2. *$x_s < \tilde{x_s}$ for some $s \in A \in \pi \Leftrightarrow x_s < \tilde{x_s}$ for all $s \in A \in \pi \Leftrightarrow V_A(x) < V_A(\tilde{x})$;*

3. *$x_s = \tilde{x_s}$ for some $s \in A \in \pi \Leftrightarrow x_s = \tilde{x_s}$ for all $s \in A \in \pi \Leftrightarrow V_A(x) = V_A(\tilde{x})$.*

Proof. Note that FOC for consumption inside an event A is the same for both consumers. Then the result follows:

$$\frac{P(s_i)}{P(s_j)} \frac{u'(x_i)}{u'(x_j)} = \frac{p_i}{p_j} \Rightarrow \frac{u'(x_i)}{u'(x_j)} = \frac{u'(\tilde{x}_i)}{u'(\tilde{x}_j)} \text{ for all } s_i, s_j \in A.$$

□

Lemma 6. *Suppose either $V_A(x) < V_B(x)$ and $V_A(\tilde{x}) \geq V_B(\tilde{x})$ or $V_A(x) = V_B(x)$ and $V_A(\tilde{x}) > V_B(\tilde{x})$, then one of the following holds:*

1. *$x_s > \tilde{x_s}$ for all $s \in A \cup B$;*

2. *$x_s < \tilde{x_s}$ for all $s \in A \cup B$.*

Proof. Note that $\frac{\phi'(V_A(x))}{\phi'(V_B(x))} > \frac{\tilde{\phi}'(V_A(\tilde{x}))}{\tilde{\phi}'(V_B(\tilde{x}))}$, hence, $\frac{u'(x_A)}{u'(x_B)} < \frac{u'(\tilde{x}_A)}{u'(\tilde{x}_B)}$. Then there are three possibilities:

1. $x_B > \tilde{x_B} \Rightarrow x_A > \tilde{x_A}$.

2. $x_B < \tilde{x_B} \Rightarrow V_A(\tilde{x}) > V_B(\tilde{x}) > V_B(x) > V_A(x)$, thus, $x_A < \tilde{x_A}$ by previous lemma.

3. $x_B = \tilde{x_B} \Rightarrow x_A > \tilde{x_A}$, then we get a contradiction.

□

Lemma 7. *Suppose $V_A(x) = V_B(x)$ and $V_A(\tilde{x}) = V_B(\tilde{x})$, then one of the following holds:*

1. *$x_s > \tilde{x_s}$ for all $s \in A \cup B$;*

2. *$x_s = \tilde{x_s}$ for all $s \in A \cup B$;*

3. *$x_s < \tilde{x_s}$ for all $s \in A \cup B$.*

The proof of Lemma 7 goes by analogy with Lemma 6.

Lemma 8. *Suppose $V_A(x) \leq V_B(x)$ and $V_A(\tilde{x}) < V_B(\tilde{x})$, then one of the following holds*

1. *$x_s \leq \tilde{x_s}$ for all $s \in A$ and $x_s < \tilde{x_s}$ for all $s \in B$;*

2. *$x_s > \tilde{x_s}$ for all $s \in A$.*

Proof. First, suppose that $x_A \leq \tilde{x_A}$, then two situations are possible.

1. $\frac{\phi'(V_A(x))}{\phi'(V_B(x))} > \frac{\tilde{\phi}'(V_A(\tilde{x}))}{\tilde{\phi}'(V_B(\tilde{x}))} \Rightarrow \frac{u'(x_A)}{u'(\tilde{x_A})} < \frac{u'(x_B)}{u'(\tilde{x_B})}$, hence, $x_B < \tilde{x_B}$.

2. $\frac{\phi'(V_A(x))}{\phi'(V_B(x))} \leq \frac{\tilde{\phi}'(V_A(\tilde{x}))}{\tilde{\phi}'(V_B(\tilde{x}))}$, however, ϕ is more concave than $\tilde{\phi}$, thus, $\frac{\phi'(V_A(x))}{\phi'(V_B(x))} > \frac{\tilde{\phi}'(V_A(x))}{\tilde{\phi}'(V_B(x))}$. It means that the situation when $V_A(x) < V_A(\tilde{x}) < V_B(\tilde{x}) < V_B(x)$ is not possible. We have already obtained that $V_A(x) < V_A(\tilde{x}) < V_B(\tilde{x})$, so $V_B(x) < V_B(\tilde{x}) \Rightarrow x_B < \tilde{x_B}$.

Now suppose that $x_A > \tilde{x_A}$. Note that the condition $\frac{\phi'(V_A(x))}{\phi'(V_B(x))} > \frac{\tilde{\phi}'(V_A(\tilde{x}))}{\tilde{\phi}'(V_B(\tilde{x}))}$ allows both situations $x_B \leq \tilde{x_B}$ and $x_B > \tilde{x_B}$, so we can make a conclusion only about x_A. ☐

Consider five types of different events for a less events-averse consumer with the following order of valuations:

$$V_P(\tilde{x}), V_T(\tilde{x}) < V_A(\tilde{x}) = V_D(\tilde{x}) < V_B(\tilde{x}), V_K(\tilde{x}).$$

Suppose that for the more events-averse consumer the values are ordered

$$V_K(x), V_T(x) < V_A(x) \leq V_D(x) < V_B(x), V_P(x).$$

Then two situations are possible:

1. Suppose that $V_A(\tilde{x}) \geq V_A(x)$, then by Lemma 6 and Lemma 7 we obtain that $V_P(\tilde{x}) \geq V_P(x)$, $V_D(\tilde{x}) \geq V_D(x)$ and $V_K(\tilde{x}) > V_K(x)$. In addition, by Lemma 8, we get $V_B(\tilde{x}) > V_B(x)$, and we know nothing about event T. Hence, if the value of a middle-value event goes down with more concavity, then the values of the events with greater values also decrease.

2. Suppose that $V_A(\tilde{x}) < V_A(x)$, then by Lemma 6 we obtain that $V_P(\tilde{x}) < V_P(x)$, $V_D(\tilde{x}) < V_D(x)$ and $V_K(\tilde{x}) < V_K(x)$. In addition, by Lemma 8, we get $V_T(\tilde{x}) > V_T(x)$, and we know nothing about event B. Hence, if the value of a middle-value event goes up with more concavity, then the values of the events with lower values also increase.

Also note that all values cannot change in the same direction as the income stays constant. From the above, we conclude that there exists a number N such that all values below it would increase with concavity and all values above it will decrease. Clearly, there is an infinite number of ways to define N. However, note that only the limit value $N(u, \pi, P, p, I)$ would satisfy all of the possible conditions arising from different functions ϕ and $\tilde{\phi}$.

A.3.2 Proof of Theorem 2

Lemma 9. *Suppose that $\pi = \{A, s\}$, where $A = \{s_1, s_2, \ldots, s_k\}$, and $\tilde{\pi} = \{A \backslash s_1, s_1, s\}$. Assume also that both $u(\cdot)$ and $\phi(\cdot)$ are differentiable concave functions. If x_s denotes consumption in state s under π, while $\tilde{x}_s$ is consumption under $\tilde{\pi}$, then one of the following holds:*

1. *$\tilde{x}_s \geq x_s$, $\tilde{x}_1 < x_1$, and $\tilde{x}_i > x_i$, where $i \neq 1$;*

2. *$\tilde{x}_s \geq x_s$, $\tilde{x}_1 > x_1$, and $\tilde{x}_i < x_i$, where $i \neq 1$;*

3. *$\tilde{x}_s < x_s$, $\tilde{x}_1 < x_1$, and $\tilde{x}_i > x_i$, where $i \neq 1$;*

4. *$\tilde{x}_s < x_s$, $\tilde{x}_1 > x_1$, and $\tilde{x}_i < x_i$, where $i \neq 1$;*

5. *$\tilde{x}_s = x_s$, $\tilde{x}_1 = x_1$, and $\tilde{x}_i = x_i$, where $i \neq 1$.*

Proof. Denote $V_A(x) = \sum_{s_i \in A} P(s_i|A)u(x_i)$, then the first-order conditions between s_1 and the other states $s_i \in A$ for the SASEU agent under π can be rewritten as

$$\frac{p_1}{p_i}\frac{P(s_i)}{P(s_1)} = \frac{u'(x_1)}{u'(x_i)}. \tag{10}$$

While the first-order conditions between s_1 and the other states in A for the SASEU agent under $\tilde{\pi}$ are

$$\frac{p_1}{p_i}\frac{P(s_i)}{P(s_1)} = \frac{\phi'(u(\tilde{x}_1))}{\phi'(V_{A\backslash s_1}(\tilde{x}))}\frac{u'(\tilde{x}_1)}{u'(\tilde{x}_i)}. \tag{11}$$

The first-order condition between outside state s and the states in A (including s_1) under π is

$$\frac{p_i}{p_s}\frac{P(s)}{P(s_i)} = \frac{\phi'(V_A(x))}{\phi'(u(x_s))}\frac{u'(x_i)}{u'(x_s)}. \tag{12}$$

The first-order condition between outside state s and s_1 under $\tilde{\pi}$ is

$$\frac{p_1}{p_s}\frac{P(s)}{P(s_1)} = \frac{\phi'(u(\tilde{x}_1))}{\phi'(u(\tilde{x}_s))}\frac{u'(\tilde{x}_1)}{u'(\tilde{x}_s)}. \tag{13}$$

The first-order condition between outside state s and the other states in A under $\tilde{\pi}$ are

$$\frac{p_i}{p_s}\frac{P(s)}{P(s_i)} = \frac{\phi'(V_{A\backslash s_1}(\tilde{x}))}{\phi'(u(\tilde{x}_s))}\frac{u'(\tilde{x}_i)}{u'(\tilde{x}_s)}. \tag{14}$$

The first-order condition between states in $A\backslash s_1$ under π and $\tilde{\pi}$ are

$$\frac{p_i}{p_j}\frac{P(s_j)}{P(s_i)} = \frac{u'(x_i)}{u'(x_j)} \text{ and } \frac{p_i}{p_j}\frac{P(s_j)}{P(s_i)} = \frac{u'(\tilde{x}_i)}{u'(\tilde{x}_j)} \tag{15}$$

$$\Rightarrow \frac{u'(x_i)}{u'(x_j)} = \frac{u'(\tilde{x}_i)}{u'(\tilde{x}_j)}. \tag{16}$$

The above conditions imply the following:

$$\frac{\phi'(u(\tilde{x}_s))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} = \frac{\phi'(V_{A\backslash s_1}(\tilde{x}))}{\phi'(V_A(x))}\frac{u'(\tilde{x}_i)}{u'(x_i)} = \frac{\phi'(u(\tilde{x}_1))}{\phi'(V_A(x))}\frac{u'(\tilde{x}_1)}{u'(x_1)}. \tag{17}$$

Now we consider different cases.

1. $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} \leq 1$ and $\frac{u'(\tilde{x}_1)}{u'(x_1)} \geq \frac{u'(\tilde{x}_i)}{u'(x_i)}$.

   First, consider $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} < 1$. Concave $u(\cdot)$ and $\phi(\cdot)$ imply that $\tilde{x}_s > x_s$. Now suppose that $\tilde{x}_1 \geq x_1$, then $1 \geq \frac{u'(\tilde{x}_1)}{u'(x_1)} \geq \frac{u'(\tilde{x}_i)}{u'(x_i)}$ implying that $\tilde{x}_i \geq x_i$. However, note that the FOC for the states in $A\backslash s_1$ guarantees that if $\tilde{x}_i \geq x_i$, then $\tilde{x}_j \geq x_j$ for all $i, j \in A\backslash s_1$. Which leads us to a contradiction, because consumption in some states cannot go up without falling in the other states if there is no change in prices or income. Hence, $\tilde{x}_1 < x_1$.

   Now we are left to show that $\tilde{x}_i > x_i$. First, note that $\frac{u'(\tilde{x}_1)}{u'(x_1)} \geq \frac{u'(\tilde{x}_i)}{u'(x_i)}$ implies $\phi'(V_{A\backslash s_1}(\tilde{x})) \geq \phi'(u(\tilde{x}_1))$, so $u(c_1) > u(\tilde{c}_1) \geq V_{A\backslash s_1}(\tilde{x})$. Then two situations are possible: (1) If $V_{A\backslash s_1}(\tilde{x}) \geq V_A(x)$, then

   $$V_A(x) = P(s_1|A)u(x_1) + (1 - P(s_1|A))V_{A\backslash s_1}(x) > P(s_1|A)V_{A\backslash s_1}(\tilde{x}) + (1 - P(s_1|A))V_{A\backslash s_1}(x)$$
   $$\Rightarrow V_{A\backslash s_1}(\tilde{x}) > V_{A\backslash s_1}(x).$$

   According to the FOC for the states in $A\backslash s_1$, consumption in all states moves in the same direction, hence, implying that $\tilde{x}_i > x_i$. (2) If $V_{A\backslash s_1}(\tilde{x}) < V_A(x)$, then $\frac{u'(\tilde{x}_i)}{u'(x_i)} < 1$ implying that $\tilde{x}_i > x_i$ anyway.

   Thus, we obtain $\tilde{x}_s > x_s$, $\tilde{x}_1 < x_1$, and $\tilde{x}_i > x_i$, where $i \neq 1$.

   Now consider $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} = 1 \Rightarrow \tilde{x}_s = x_s$. In this situation, $\frac{u'(\tilde{x}_1)}{u'(x_1)} \geq \frac{u'(\tilde{x}_i)}{u'(x_i)}$ implies that either (1) $\tilde{x}_1 < x_1$, and $\tilde{x}_i > x_i$; or (2) $\tilde{x}_1 = x_1$, and $\tilde{x}_i = x_i$. Hence, we obtain cases 1 and 5 in the lemma.

2. $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} \leq 1$ and $\frac{u'(\tilde{x}_1)}{u'(x_1)} < \frac{u'(\tilde{x}_i)}{u'(x_i)}$.

   First, consider $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} < 1$. Concave $u(\cdot)$ and $\phi(\cdot)$ imply that $\tilde{x}_s > c_s$. Now suppose that $\tilde{x}_i \geq x_i$, then $\frac{u'(\tilde{x}_1)}{u'(x_1)} < \frac{u'(\tilde{x}_i)}{u'(x_i)} \leq 1$ implying that $\tilde{x}_1 > x_1$ and $\tilde{x}_i > x_i$. However, note that the FOC for the states in $A\backslash s_1$ guarantees that if $\tilde{x}_i > x_i$, then $\tilde{x}_j > x_j$ for all $i, j \in A\backslash s_1$. Which leads us to a contradiction, because consumption in all states cannot go up without the change in prices or income. Hence, $\tilde{x}_i < x_i$ for all $s_i \in A\backslash s_1$.

Now we are left to show that $\tilde{x}_1 > x_1$. First, note that $\frac{u'(\tilde{x}_i)}{u'(x_i)} > 1$ implies $\phi'(V_{A\backslash s_1}(\tilde{x})) < \phi'(V_A(x))$, so $V_{A\backslash s_1}() > V_{A\backslash s_1}(\tilde{x}) > V_A(x) \Rightarrow u(x_1) < V_{A\backslash s_1}(x)$. Then two situations are possible: (1) If $u(\tilde{x}_1) > V_A(x)$, then

$$V_A(x) = P(s_1|A)u(x_1) + (1 - P(s_1|A))V_{A\backslash s_1}(x) > u(x_1)$$
$$\Rightarrow u(\tilde{x}_1) > u(x_1).$$

(2) If $u(\tilde{x}_1) < V_A(x)$, then $\frac{u'(\tilde{x}_1)}{u'(x_1)} < 1$ implying that $\tilde{x}_1 > x_1$ anyway.

Thus, we obtain $\tilde{x}_s > x_s$, $\tilde{x}_1 > x_1$, and $\tilde{x}_i < x_i$, where $i \neq 1$.

Now consider $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} = 1 \Rightarrow \tilde{x}_s = x_s$. Taking into account $\frac{u'(\tilde{x}_1)}{u'(x_1)} < \frac{u'(\tilde{x}_i)}{u'(x_i)}$, we obtain $\tilde{x}_1 > x_1$, and $\tilde{x}_i < x_i$. Hence, we get case 2 in the lemma.

3. $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} > 1$ and $\frac{u'(\tilde{x}_1)}{u'(x_1)} \geq \frac{u'(\tilde{x}_i)}{u'(x_i)}$.

   Concave $u(\cdot)$ and $\phi(\cdot)$ imply that $\tilde{x}_s < x_s$. Now suppose that $\tilde{x}_i \leq x_i$, then $\frac{u'(\tilde{x}_1)}{u'(x_1)} \geq \frac{u'(\tilde{x}_i)}{u'(x_i)} \geq 1$ implying that $\tilde{x}_1 \leq x_1$. However, note that the FOC for the states in $A\backslash s_1$ guarantees that if $\tilde{x}_i \leq x_i$, then $\tilde{x}_j \leq x_j$ for all $i, j \in A\backslash s_1$. Which leads us to a contradiction, because consumption in several states cannot go down without going up at least in one other state if there is no change in prices or income. Hence, $\tilde{x}_i > x_i$ for all $s_i \in A\backslash s_1$.

   Now we are left to show that $\tilde{x}_1 < x_1$. First, note that $\frac{u'(\tilde{x}_i)}{u'(x_i)} < 1$ implies $\phi'(V_{A\backslash s_1}(\tilde{x})) > \phi'(V_A(x))$, so $V_{A\backslash s_1}(x) < V_{A\backslash s_1}(\tilde{x}) < V_A(x) \Rightarrow u(x_1) > V_{A\backslash s_1}(x) \Rightarrow u(x_1) > V_A(x)$. Then two situations are possible: (1) If $u(\tilde{x}_1) \leq V_A(x)$, then by taking into account $u(x_1) > V_A(x) \Rightarrow u(x_1) > u(\tilde{x}_1)$; (2) If $u(\tilde{x}_1) > V_A(x)$, then $\frac{u'(\tilde{c}_1)}{u'(c_1)} > 1$ implies that $\tilde{x}_1 < x_1$ anyway.

   Thus, we obtain case 3 in the lemma, where $\tilde{x}_s < x_s$, $\tilde{x}_1 < x_1$, and $\tilde{x}_i > x_i$, where $i \neq 1$.

4. $\frac{\phi'(u(\tilde{x_s}))}{\phi'(u(x_s))}\frac{u'(\tilde{x}_s)}{u'(x_s)} > 1$ and $\frac{u'(\tilde{x}_1)}{u'(x_1)} < \frac{u'(\tilde{x}_i)}{u'(x_i)}$.

   Concave $u(\cdot)$ and $\phi(\cdot)$ imply that $\tilde{x}_s < x_s$. Now suppose that $\tilde{x}_1 \leq x_1$, then $1 \leq \frac{u'(\tilde{x}_1)}{u'(x_1)} < \frac{u'(\tilde{x}_i)}{u'(x_i)}$ implying that $\tilde{x}_1 \leq x_1$ and $\tilde{x}_i < x_i$. However, note that the FOC for the states in $A \backslash s_1$ guarantees that if $\tilde{x}_i < x_i$, then $\tilde{x}_j < x_j$ for all $i, j \in A \backslash s_1$. Which leads us to a contradiction, because consumption in all states cannot go down without the change in prices or income. Hence, $\tilde{x}_1 > x_1$.

Now we are left to show that $\tilde{x}_i < x_i$. First, note that $\frac{u'(\tilde{x}_1)}{u'(x_1)} < \frac{u'(\tilde{x}_i)}{u'(x_i)}$ implies $\phi'(V_{A\backslash s_1}(\tilde{x})) < \phi'(u(\tilde{x}_1))$, so $u(x_1) < u(\tilde{x}_1) < V_{A\backslash s_1}(\tilde{x})$. Then two situations are possible: (1) If $V_{A\backslash s_1}(\tilde{x}) < V_A(x)$, then

$$V_A(x) = P(s_1|A)u(x_1) + (1 - P(s_1|A))V_{A\backslash s_1}(x) < P(s_1|A)V_{A\backslash s_1}(\tilde{x}) + (1 - P(s_1|A))V_{A\backslash s_1}(x)$$

$$\Rightarrow V_{A\backslash s_1}(\tilde{x}) < V_{A\backslash s_1}(x).$$

According to the FOC for the states in $A \backslash s_1$, consumption in all states moves in the same direction, hence, implying that $\tilde{x}_i < x_i$. (2) If $V_{A\backslash s_1}(\tilde{x}) > V_A(x)$, then $\frac{u'(\tilde{x}_i)}{u'(x_i)} > 1$ implying that $\tilde{x}_i < x_i$ anyway.

Thus, we obtain case 4 in the lemma, where $\tilde{x}_s < x_s$, $\tilde{x}_1 > x_1$, and $\tilde{x}_i < x_i$, where $i \neq 1$.

□

Lemma 10. *Suppose that* $\pi = \{A_1, A_2, \ldots, A_k\}$ *and* $\tilde{\pi} = \{A_1 \backslash s_1, s_1, A_2, \ldots, A_k\}$. *Assume also a concave and differentiable* $u(\cdot)$ *and* $\phi(\cdot)$. *Then for all events* A_i *such that* $i \neq 1$ *one of the following holds:*

1. $\tilde{x}_s > x_s$ *for all* $s \in A_i$*;*

2. $\tilde{x}_s < x_s$ *for all* $s \in A_i$*;*

3. $\tilde{x}_s = x_s$ *for all* $s \in A_i$*.*

Proof. Consider the first-order condition between states s_i and s_j inside event A_i, noting that it is not affected by disaggregation of s_1:

$$\frac{u'(x_j)}{u'(x_i)} = \frac{p_j}{p_i}\frac{P(s_i)}{P(s_j)} = \frac{u'(\tilde{x}_j)}{u'(\tilde{x}_i)}.$$

Thus, if $\tilde{x}_j > x_j$, then $\tilde{x}_i > x_i$, and vice versa. Given that the condition is the same for all states in A_i, we obtain that consumption in all states inside A_i moves in the same direction.

Now consider two different states from different events $s_i \in A_i$ and $s_j \in A_j$ that are unaffected by the change in state aggregation. Note that $\tilde{x}_j > x_j \Leftrightarrow V_{A_j}(\tilde{x}) > V_{A_j}(x)$ for any $s_j \in A_j$, where $j \neq 1$. By taking into account the first-order condition between the states from different events

$$\frac{\phi'(V_{A_j}(x))u'(x_j)}{\phi'(V_{A_i}(x))u'(x_i)} = \frac{p_j}{p_i}\frac{P(s_i)}{P(s_j)} = \frac{\phi'(V_{A_j}(\tilde{x}))u'(\tilde{x}_j)}{\phi'(V_{A_i}(\tilde{x}))u'(\tilde{x}_i)},$$

we obtain that $V_{A_j}(\tilde{x}) > V_{A_j}(x) \Leftrightarrow V_{A_i}(\tilde{x}) > V_{A_i}(x)$, and the result follows. □

Lemma 11. *Consider two partitions π and $\tilde{\pi}$ such that $\tilde{\pi} = \{A_1\backslash s_1, s_1, A_2, \dots, A_k\}$ and $\pi = \{A_1, A_2, \dots, A_k\}$. In addition, suppose that both $u(\cdot)$ and $\phi(\cdot)$ are concave and differentiable. Then*

1. *$u(x_1) > V_{A\backslash s_1}(x) \Leftrightarrow u(\tilde{x}_1) > V_{A\backslash s_1}(\tilde{x}) \Leftrightarrow \tilde{x}_1 < x_1, \tilde{x}_i > x_i$ for any $s_i \in A\backslash s_1$;*
2. *$u(x_1) < V_{A\backslash s_1}(x) \Leftrightarrow u(\tilde{x}_1) < V_{A\backslash s_1}(\tilde{x}) \Leftrightarrow \tilde{x}_1 > x_1, \tilde{x}_i < x_i$ for any $s_i \in A\backslash s_1$;*
3. *$u(x_1) = V_{A\backslash s_1}(x) \Leftrightarrow u(\tilde{x}_1) = V_{A\backslash s_1}(\tilde{x}) \Leftrightarrow \tilde{x}_1 = x_1, \tilde{x}_i = x_i$, $\tilde{x}_s = x_s$ for any $s_i \in A\backslash s_1$, $s \in A_k$ and all $k \neq 1$.*

Moreover, if 1 or 2 is true, then one of the following holds:

1. *$\tilde{x}_s > x_s$ for all $s \in A_k$ and all $k \neq 1$;*
2. *$\tilde{x}_s < x_s$ for all $s \in A_k$ and all $k \neq 1$;*
3. *$\tilde{x}_s = x_s$ for all $s \in A_k$ and all $k \neq 1$.*

Proof. The last lemma implies that consumption in unaffected events moves in the same direction, then together with Lemma 9, the proof of this lemma follows. □

The proof of the theorem follows from the fact that consumption in the states in B moves in the same direction, because FOC between these states is not affected. It implies that $V_B(x)$ moves together with consumption in any of the states in B. By using this result and previous lemma, the rest follows.

A.4 Identification

A.4.1 Proof of Theorem 3

The agent purchases a bundle of Arrow securities that maximizes her value given a certain amount of income I and the price p_i of an Arrow security that pays 1 in state i:

$$\max_x V_\pi(x)$$
$$\text{s.t.} \quad \sum_i p_i x_i = I.$$

Hence, if λ is a Lagrange multiplier, then the first-order condition for each state $s \in A$ is

$$p_A \phi' \left(\sum_{s \in A} P(s|A) u(x_s) \right) p(s|A) u'(x_s) = \lambda p_s^x.$$

Pick two states s_i and s_j.

Two states from the same event: If s_i and s_j belong to the same event A, then

$$\frac{p(s_i|A)}{p(s_j|A)} \frac{u'(x_{s_i})}{u'(x_{s_j})} = \frac{p_{s_i}^x}{p_{s_j}^x}. \tag{18}$$

Now choose two other bundles y and z such that $y_{s_i} = x_{s_i}$, $z_{s_j} = x_{s_j}$, but $y_{s_j} \neq x_{s_j}$ and $z_{s_i} \neq x_{s_i}$. Then we obtain

$$\frac{u'(y_{s_j})}{u'(x_{s_j})} = \frac{p_{s_i}^x}{p_{s_j}^x} \frac{p_{s_j}^y}{p_{s_i}^y} \text{ and } \frac{u'(x_{s_i})}{u'(z_{s_i})} = \frac{p_{s_i}^x}{p_{s_j}^x} \frac{p_{s_j}^z}{p_{s_i}^z}. \tag{19}$$

Note that both left-side ratios depend only on payoffs at a related state and nothing else. Thus, if payoffs at other states are changed, it must not affect the above ratios.

Two states from different events: We repeat the above derivation when states s_i and s_j are from different events A_i and A_j:

$$\frac{p_{A_i} P(s_i|A_i)}{p_{A_j} p(s_j|A_j)} \frac{\phi' \left(\sum_{s \in A_i} P(s|A_i) u(x_s) \right) u'(x_{s_i})}{\phi' \left(\sum_{s \in A_j} P(s|A_j) u(x_s) \right) u'(x_{s_j})} = \frac{p_{s_i}^x}{p_{s_j}^x}. \tag{20}$$

After taking bundles y and z as before, we get

$$\frac{\phi'\left(\sum_{s\in A_j} P(s|A_j)u(y_s)\right)\phi'\left(\sum_{s\in A_i} P(s|A_i)u(x_s)\right)}{\phi'\left(\sum_{s\in A_i} P(s|A_i)u(y_s)\right)\phi'\left(\sum_{s\in A_j} P(s|A_j)u(x_s)\right)}\frac{u'(y_{s_j})}{u'(x_{s_j})} = \frac{p^x_{s_i}}{p^x_{s_j}}\frac{p^y_{s_j}}{p^y_{s_i}} \tag{21}$$

$$\frac{\phi'\left(\sum_{s\in A_j} P(s|A_j)u(z_s)\right)\phi'\left(\sum_{s\in A_i} P(s|A_i)u(x_s)\right)}{\phi'\left(\sum_{s\in A_i} P(s|A_i)u(z_s)\right)\phi'\left(\sum_{s\in A_j} P(s|A_j)u(x_s)\right)}\frac{u'(x_{s_i})}{u'(z_{s_i})} = \frac{p^x_{s_i}}{p^x_{s_j}}\frac{p^z_{s_j}}{p^z_{s_i}}. \tag{22}$$

Notice that the left sides of *both* equalities do not depend on other states if and only if both s_i and s_j are singleton events. Thus, we might get confused and aggregate all singletons together. However, consider FOC between a singleton s_i and a state s_j from a non-singleton event A_j:

$$\frac{\phi'\left(\sum_{s\in A_j} P(s|A_j)u(y_s)\right)\phi'\left(u(x_{s_i})\right)}{\phi'\left(u(y_{s_i})\right)\phi'\left(\sum_{s\in A_j} P(s|A_j)u(x_s)\right)}\frac{u'(y_{s_j})}{u'(x_{s_j})} = \frac{p^x_{s_i}}{p^x_{s_j}}\frac{p^y_{s_j}}{p^y_{s_i}}. \tag{23}$$

Note that the left side depends on payoffs in all states at A_j and s_i, however, it does not depend on the payoffs at other singletons. Hence, after recognizing the groups of the states that might be potential events, if there are two or more of such groups, we are able to identify which group is the group of the singletons. Otherwise, if there is only one group, then we will never be able to distinguish between Ω and $\{\Omega\}$ without additional information. However, Assumption 2 restricts us to Ω. Thus, the state aggregation π is identified.

Now note that by choosing different values of x_{s_i}, x_{s_j}, y_{s_j} and z_{s_i} when s_i and s_j belong to the same event, we can identify $u(\cdot)$ up to affine transformation. After that we consider the original first-order condition again and identify the probability ratios $\frac{P(s_i|A)}{P(s_j|A)}$. Given that $\sum_{s|A} P(s|A) = 1$, we can identify the probabilities.

To identify $\phi(\cdot)$, we consider states from different events: $u(\cdot)$ and all conditional probabilities have already been identified, hence, we can obtain the values of different ratios of the kind

$$\frac{\phi'\left(\sum_{s\in A_j} P(s|A_j)u(y_s)\right)\phi'\left(\sum_{s\in A_i} P(s|A_i)u(x_s)\right)}{\phi'\left(\sum_{s\in A_i} P(s|A_i)u(y_s)\right)\phi'\left(\sum_{s\in A_j} P(s|A_j)u(x_s)\right)}.$$

Note that if A_i is a singleton and y_{s_i} was chosen such that $y_{s_i} = x_{s_i}$, then $\phi'\left(\sum_{s \in A_i} P(s|A_i)u(x_s)\right)$ and $\phi'\left(\sum_{s \in A_i} P(s|A_i)u(y_s)\right)$ cancel each other out. If A_i is not a singleton, $\phi'\left(\sum_{s \in A_i} P(s|A_i)u(x_s)\right)$ and $\phi'\left(\sum_{s \in A_i} P(s|A_i)u(y_s)\right)$ can be canceled out by choosing $y_s = x_s$ for all $s \in A_i$. Hence, we can identify $\phi(\cdot)$ up to affine transformation.

The only unknown variables left are priors over events p_{A_i}. However, we can obtain $\frac{p_{A_i}}{p_{A_j}}$ from the first-order condition for two states from different events (see (11)). Finally, given that $\sum_i p_{A_i} = 1$, we identify p_{A_i} as well.

A.4.2 Generalization

Theorem 6. *Consider a finite sequence of finer partitions $\pi_1, \ldots, \pi_k$ of Ω. Suppose the agent does several rounds of state aggregation according to $\pi_1, \ldots, \pi_k$ with concave curvature functions $\phi_1, \ldots, \phi_k$. Denote A^i an event in partition π^i. If Assumptions 1 and 2 hold, then partitions $\pi_1, \ldots, \pi_k$ and corresponding conditional probabilities are identified. Moreover, utility function u and curvature functions $\phi_1, \ldots, \phi_k$ are identified up to affine transformation.*

Proof. By following the procedure from Theorem 3, we easily identify the finest partition π_k, utility u, and conditional probabilities $P(s|A^k)$. Next, we construct values at the events of partition π^k: $V_{A^k}(x) = \sum_{s \in A^k} P(s|A^k)u(x_s)$. After that we repeat the entire procedure again, but this time using $V_{A^k}(x)$ instead of consumption x. That would allow us to identify π^{k-1}, ϕ^k and probabilities $P(A^k|A^{k-1})$. Then we construct the values $V_{A^{k-1}}(x) = \sum_{A^k \in A^{k-1}} P(A^k|A^{k-1})V_{A^k}(x)$ and repeat the procedure again and again until we reach π_1. □